RIETI Discussion Paper Series 13-E-089

Synchronization and the Coupled Oscillator Model in International Business Cycles

IKEDA Yuichi
Kyoto University

AOYAMA Hideaki
RIETI

YOSHIKAWA Hiroshi
RIETI

Research Institute of Economy, Trade & Industry, IAA

The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 13-E-089

October 2013

Synchronization and the Coupled Oscillator Model in International Business Cycles

IKEDA Yuichi
Graduate School of Advanced Integrated Studies in Human Survivability, Kyoto University
AOYAMA Hideaki
Graduate School of Science, Kyoto University and RIETI
YOSHIKAWA Hiroshi
Graduate School of Economics, University of Tokyo and RIETI

Abstract

Synchronization in international business cycles attracts economists and physicists as an example of self-organization in the time domain. In economics, synchronization of the business cycles has been discussed using correlation coefficients between gross domestic product (GDP) time series. However, more definitive discussions using a suitable quantity describing the business cycles are needed. In this paper, we analyze the quarterly GDP time series for Australia, Canada, France, Italy, the United Kingdom, and the United States from Q2 1960 to Q1 2010 in order to obtain direct evidence for the synchronization and to clarify its origin. We find frequency entrainment and partial phase locking to be direct evidence of synchronization in international business cycles. Furthermore, a coupled limit-cycle oscillator model is developed to explain the mechanism of synchronization. In this model, the interaction due to international trade is interpreted as the origin of the synchronization.

Keywords: Business cycle, Synchronization, and Hilbert transform
JEL classification: Macroeconomics and Monetary Economics

This study is conducted as a part of the Project "Dynamics, Energy and Environment, and Growth of Small-and Medium-sized Enterprises" undertaken at Research Institute of Economy, Trade and Industry (RIETI). The author is grateful for helpful comments and suggestions by H. Iyetomi, Y. Fujiwara, W. Souma and Discussion Paper seminar participants at RIETI.

1 Introduction

Business cycles have a long history of being subjected to theoretical studies [1, 2, 3]. Synchronization [4] in the international business cycles in particular attracts economists and physicists as an example of self-organization in the time domain [5]. Synchronization of business cycles across countries has been discussed using correlation coefficients between GDP time series [6]. However, this method remains only a primitive first-step, and more definitive analysis using a suitable quantity describing the business cycles is needed.

We analyze the quarterly GDP time series for Australia, Canada, France, Italy, the United Kingdom, and the United States. The purpose of studying the international business cycles is to answer the following questions:

(i) Can we obtain direct evidence for the synchronization in business cycles?

(ii) If so, what is the mechanism causing such synchronization?

(iii) In relation to question (ii), what types of economic shocks play an important role in business cycles?

(iv) What is the economic origin of the synchronization?

In analyzing business cycles, an important question is the significance of individual (micro) versus aggregate (macro) shocks. Foerster et. al. [7], using factor analysis, showed that the volatility of the United States industrial production was largely explained by aggregate shocks, and partly by cross-sectoral correlation due to the individual shocks transformed through the trade linkage. We take a different approach to analyze the shocks in explaining the synchronization in the international business cycles in this paper.

The rest of the paper is organized as follows. In section 2, empirical analysis of the GDP time series for the six countries using the Hilbert transform is explained. In section 3, we show frequency entrainment and phase locking as evidence of synchronization in the international business cycles. We then discuss common shocks (comovement) and individual shocks using the random matrix theory. Finally, we show that the origin of the observed synchronization is interaction due to international trade using a limit-cycle coupled oscillator model. The conclusions are given in section 4.

2 Empirical Analysis

2.1 Data

We analyze the quarterly GDP time series (OECD Quarterly National Accounts, QNA) for Australia, Canada, France, Italy, the United Kingdom, and the United

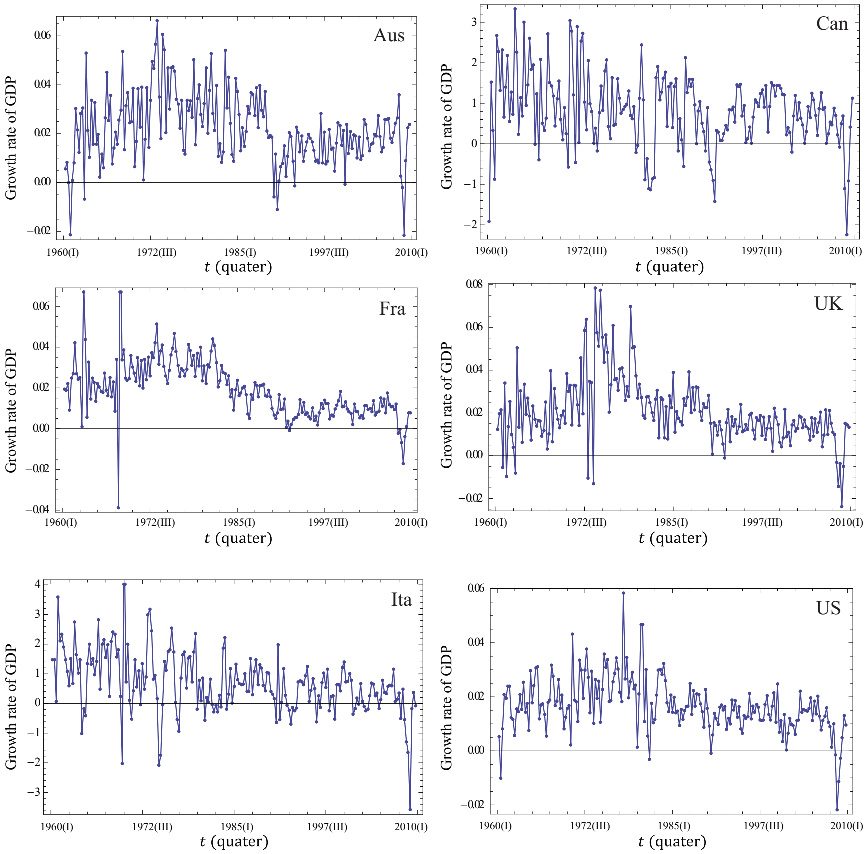

Figure 1: Growth Rate of GDP

States from Q2 1960 to Q1 2010 to study the synchronization in the international business cycles. Extracting a trend component is the important pre-processing step of the time series analysis. First, the growth rate of the GDP $x_i(t)$ defined as $x_i(t) = (GDP_i(t) - GDP_i(t-1))/GDP_i(t-1)$ were calculated for the six countries. The time series $x_i(t)$ for the six countries are shown in Fig. 1. Fourier series expansion of the time series $x_i(t)$ were then calculated. Given the identified business cycle periods of the analyzed countries, the high and low frequency Fourier components were removed, and the Fourier components the period of two to 10 years remained. The band-pass filtering of the growth rate of the GDP time series for the six countries is shown in Fig. 2.

2.2 Limit Cycle

Business cycles with a period of four to six years are usually considered to be caused by adjustments in stock, such as inventory stock. Band-pass filter was applied to the

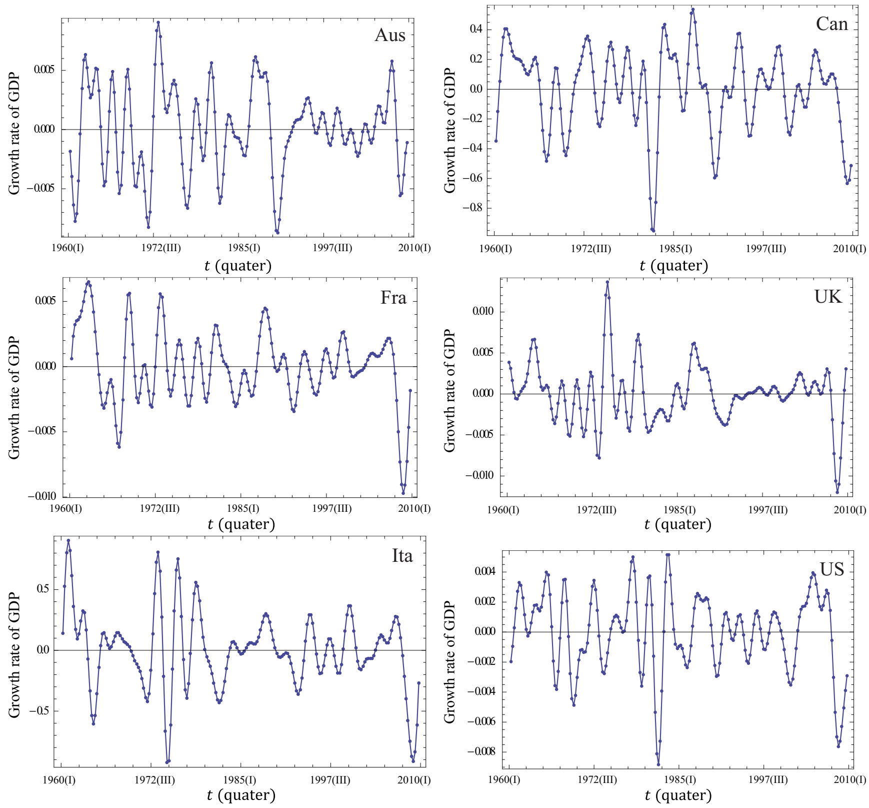

Figure 2: Filtered Growth Rate of GDP

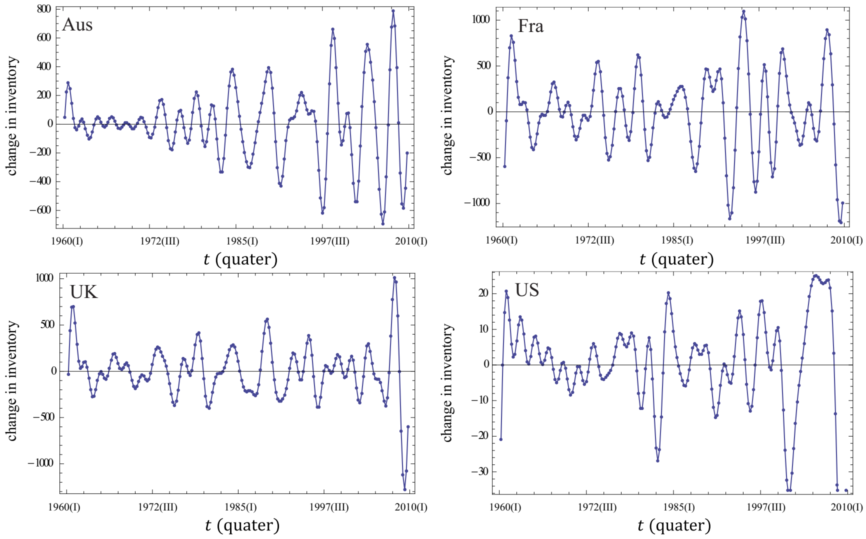

Figure 3: Filtered Change in Inventory Stock

time series of inventory changes to remove high and low frequency components, and components from the period of three to eight year remained. Frequency components were chosen for better visibility of the cycling trajectory. The obtained time series are shown for the six countries in Fig. 3.

Figure 4 depicts trajectories in the two-dimensional plane of the GDP growth rate and the changes in inventory. These commonly used figures suggest the existence of a limit-cycle in business cycles.

2.3 Hilbert Transform

The Hilbert transform is a method for analyzing the correlation of two time series with a lead-lag time relationship. The Hilbert transform of a time series $x_i(t)$ is defined by,

$$y_i(t) = H[x_i(t)] = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x_i(s)}{t-s} ds, \tag{1}$$

where PV represents the Cauchy principal value [8]. Complex time series $g_i(t)$ is obtained by adopting time series $y_i(t)$ as an imaginary part. Consequently, phase time series $\theta_i(t)$ is obtained,

$$g_i(t) = x_i(t) + iy_i(t) = A_i(t) \exp[i\theta_i(t)]. \tag{2}$$

The following example may help the readers to understand the concept of the Hilbert transform. Suppose, time series $x_i(t)$ is a cosine function $x_i(t) = \cos(\omega_i t)$, then the Hilbert transform of $x_i(t)$ will be $y_i(t) = H[\cos(\omega_i t)] = \sin(\omega_i t)$. Similarly, for a

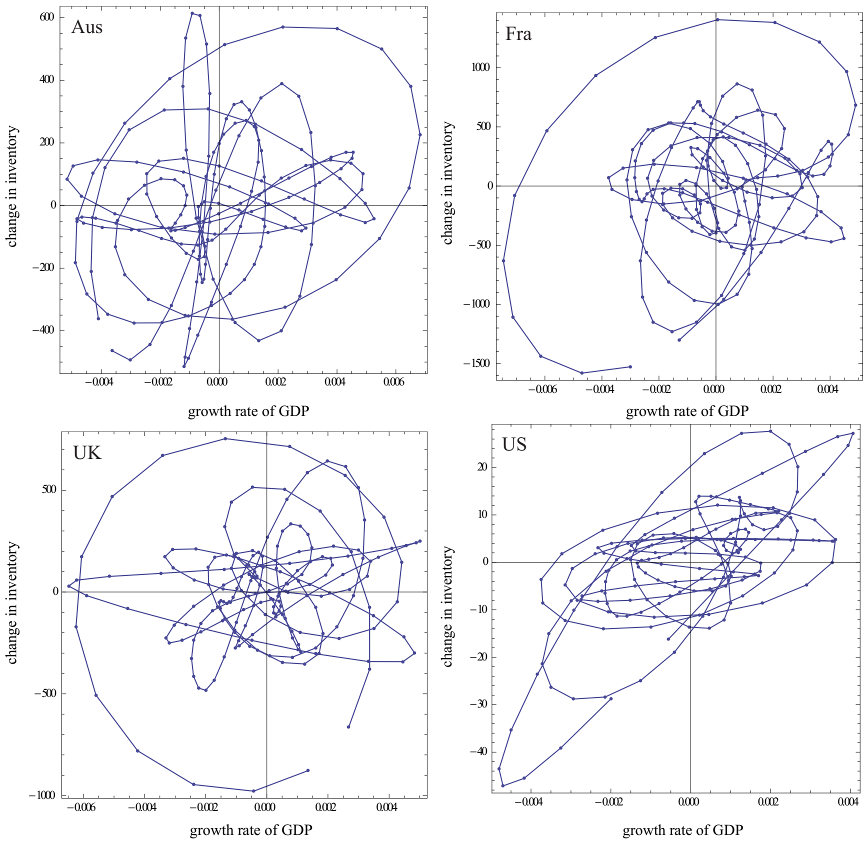

Figure 4: Growth Rate of GDP vs Change in Inventory Stock

sine function $x_i(t) = \sin(\omega_i t)$, the Hilbert transform will be $y_i(t) = H[\sin(\omega_i t)] = -\cos(\omega_i t)$. Using Euler's formula $g_i(t) = \cos(\omega_i t) + \sin(\omega_i t) = A_i(t)\exp[i\theta_i(t)]$, we obtain phase time series $\theta_i(t)$. Note that a simple time-domain correlation fails to capture the regular cycles of cosine and sine functions although they move together with an angular difference of $\pi/2$. The Hilbert transform is meant to solve the difficulty of such simple correlations.

Time series $x_i(t)$ and the Hilbert transform $y_i(t) = H[x_i(t)]$ are used as the horizontal axis and the vertical axis in the complex plane, respectively. Here, time series $x_i(t)$ is expanded as Fourier time series,

$$x_i(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left(A_n \cos\frac{n\pi t}{T} + B_n \sin\frac{n\pi t}{T} \right). \tag{3}$$

Time series $y_i(t)$ is then calculated using the Fourier coefficient in Eq.(3).

$$\begin{aligned} y_i(t) &= \frac{A_0}{2} + \sum_{n=1}^{\infty} \left(A_n H\left[\cos\frac{n\pi t}{T}\right] + B_n H\left[\sin\frac{n\pi t}{T}\right] \right) \\ &= \frac{A_0}{2} + \sum_{n=1}^{\infty} \left(A_n \sin\frac{n\pi t}{T} - B_n \cos\frac{n\pi t}{T} \right). \end{aligned} \tag{4}$$

Figure 5 depicts the obtained trajectories in the complex plane. Fourier components of oscillation for the period from two to 10 years were included in graphs of Fig. 5. Some irregular rotational movement was observed due to the non-periodic nature of the business cycles.

The time series of phase $\theta_i(t)$ was obtained using Eq.(2) for those six countries, and is depicted in Fig. 6. Fourier components of oscillation for the period from two to 10 years were included in these plots. We observed the linear trend of the phase development with some fluctuations for the six countries. The small jumps in phases in Fig. 6 were caused by the irregular rotational movement, especially the trajectories that passed near the origin of the plane, which is observed in Fig. 5.

3 Results and Discussion

3.1 Frequency Entrainment

Frequency entrainment and phase locking are expected to be observed as direct evidence of the synchronization. Angular frequency ω_i and intercept $\tilde{\theta}_i$ are estimated by fitting the time series of the phase $\theta_i(t)$ using the relation,

$$\theta_i(t) = \omega_i t + \tilde{\theta}_i, \tag{5}$$

where i indicates a country. The estimated angular frequencies ω_i for all the six countries are plotted in Fig. 7. We observe that the estimated angular frequencies

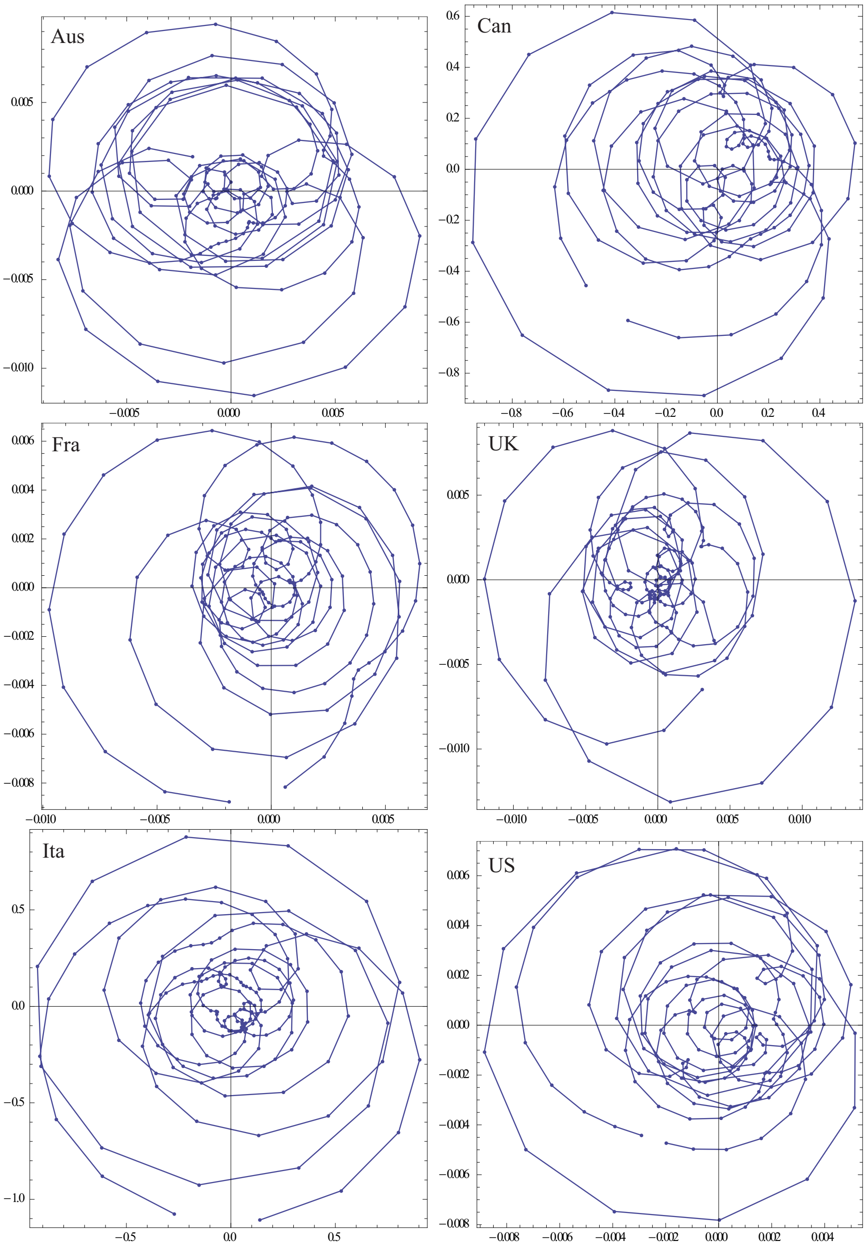

Figure 5: Trajectory in the Complex Plane

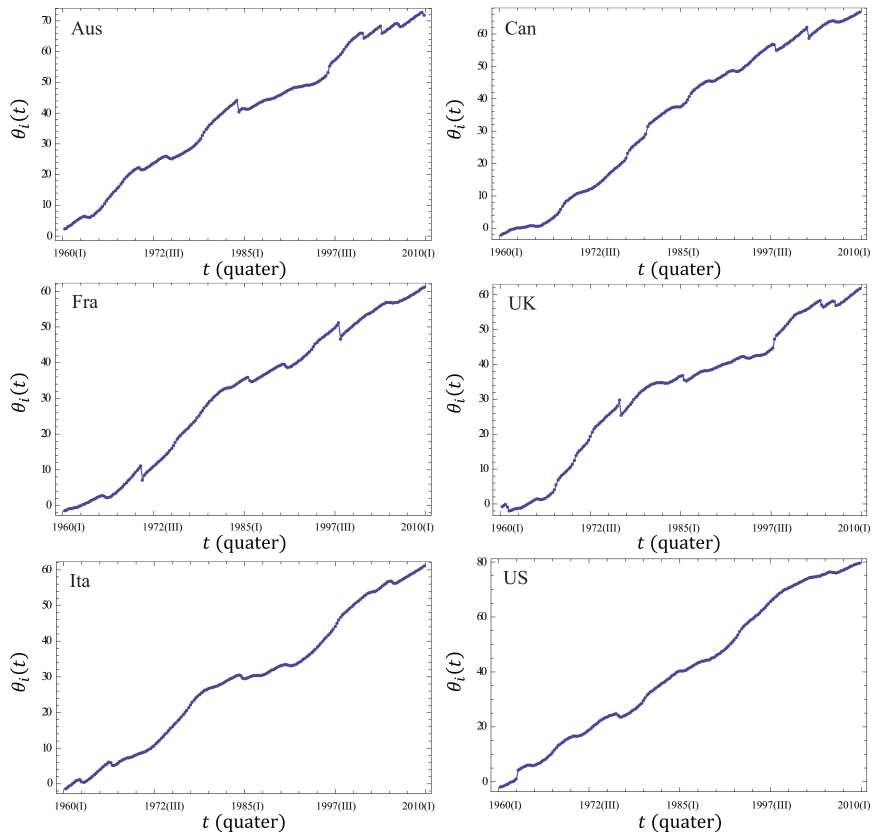

Figure 6: Time-Series of Phase obtained using Hilbert Transform

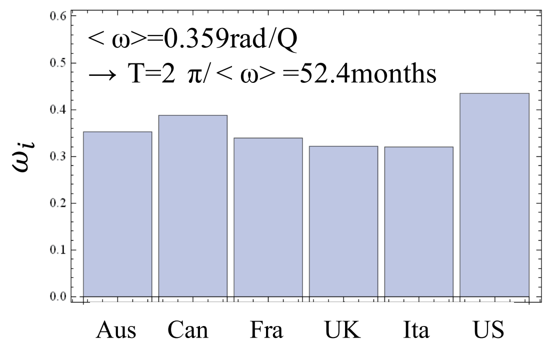

Figure 7: The Estimated Angular Frequencies

ω_i are almost identical for the six countries. This means that frequency entrainment is observed.

3.2 Phase Locking

Phase locking is the condition in which phase differences for all pairs of oscillators are constant. However, this is rarely seen to satisfy precisly the actual time series due to irregular fluctuations, i.e., economic shocks. Therefore, we introduce an indicator $\sigma(t)$ of the phase locking as

$$\sigma(t) = \Big[\frac{1}{N}\sum_{i=1}^{N}\Big\{\frac{d}{dt}(\theta_i(t) - \omega_i t) - \mu(t)\Big\}^2\Big]^{1/2}, \tag{6}$$

$$\mu(t) = \frac{1}{N}\sum_{i=1}^{N}\frac{d}{dt}(\theta_i(t) - \omega_i t). \tag{7}$$

Indicator $\sigma(t)$ is equal to zero when the phase differences for all pairs of oscillators are constant. On the other hand, if indicator $\sigma(t)$ satisfies the following relation, it is known as partial phase locking.

$$\sigma(t) \ll \omega_i, \tag{8}$$

The estimated indicator of phase locking $\sigma(t)$ is plotted in Fig. 8, which shows that indicator $\sigma(t)$ is much smaller than ω_i for most of the period. This means that the partial phase locking is observed. As a result, both frequency entrainment and phase locking are obtained as direct evidence of the synchronization.

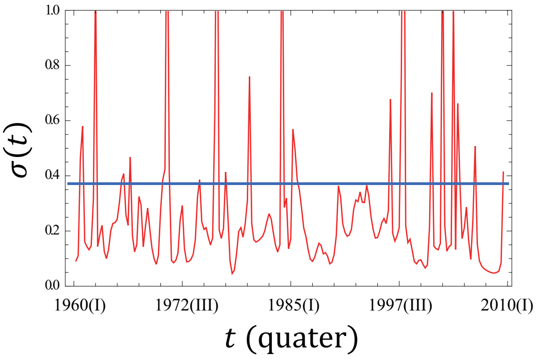

Figure 8: The Estimated Indicator of the Phase Locking

3.3 Common Shocks versus Individual Shocks

Time series $x_i(t)$ is decomposed to amplitude $A_i(t)$ and phase $\theta_i(t)$ using Eq.(1) and Eq.(2). It is interesting to ask the question "Which quantity carries information about the economic shock, amplitude $A_i(t)$ or phase $\theta_i(t)$?". The averages of these quantities over the six countries are written as:

$$\langle A(t)\rangle = \frac{1}{N}\sum_{i=1}^{N} A_i(t) = \frac{1}{N}\sum_{i=1}^{N} \frac{x_i(t)}{\cos\theta_i(t)}, \tag{9}$$

$$\langle \cos\theta(t)\rangle = \frac{1}{N}\sum_{i=1}^{N} \cos\theta_i(t). \tag{10}$$

The average amplitudes $\langle A(t)\rangle$ and the average phases $\langle \cos\theta(t)\rangle$ are shown in Fig. 9.

In the United States, we experienced eight recessions after 1960: Q1 1961, Q4 1970, Q1 1975, Q3 1980, Q4 1982, Q1 1991, Q4 2001, and Q2 2009. The recessions in 2001 and 2009 were due to the bursting of the information technology bubble and the collapse of Lehman Brothers, respectively. The value of the average amplitudes $\langle A(t)\rangle$ in Fig. 9 are large in 1961, 1975, 1982, and 2009. On the contrary, the average phases $\langle \cos\theta(t)\rangle$ in Fig. 9 show a sharp drop in all of the eight recessions described above. Therefore, we conclude that the key to understanding business cycles is phase $\theta_i(t)$, not by amplitude $A_i(t)$.

We focus on phase $\theta_i(t)$ in order to extract the common shocks (comovement, or synchronization of shocks) of the business cycles for the six countries. For this purpose, we analyzed time series $z_i(t) = cos\theta_i(t)$ using the random matrix theory [9, 10, 11, 12]. We consider the eigen-value problem

$$C|\alpha\rangle = \lambda_\alpha|\alpha\rangle, \tag{11}$$

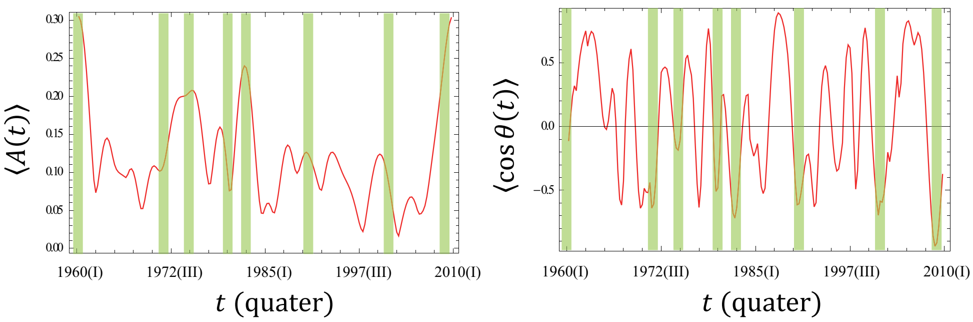

Figure 9: Average Amplitude and Phase

where λ_α and $|\alpha\rangle$ are the eigen-value and the corresponding eigen-vector, respectively, for the correlation matrix C, whose element is the correlation coefficient between countries i and j and is calculated by

$$C_{ij} = \frac{\langle (z_i(t) - \langle z_i \rangle)(z_j(t) - \langle z_j \rangle) \rangle}{\sqrt{(\langle z_i^2 \rangle - \langle z_i \rangle^2)(\langle z_j^2 \rangle - \langle z_j \rangle^2)}}, \tag{12}$$

where $\langle \cdot \rangle$ indicates the time average for time series.

We assume that the eigen-values are arranged in decreasing order ($\alpha = 0, \cdots, N-1$). Once the eigen-values are calculated using Eqs. (12) and (11), the distribution of eigen-value $\rho(\lambda)^E$ is obtained.

According to the random matrix theory, the distribution of the eigen-value for the matrix $\frac{1}{T} H H^T$, where all elements of the matrix H are given as a random number $N(0, \sigma^2)$, is given by

$$\rho(\lambda)^T = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda}, \tag{13}$$

where

$$Q = \frac{T}{N}, \tag{14}$$

$$\lambda = [\lambda_{min}, \lambda_{max}], \tag{15}$$

$$\lambda_{min} = (1 - \frac{1}{\sqrt{Q}})^2, \tag{16}$$

$$\lambda_{max} = (1 + \frac{1}{\sqrt{Q}})^2. \tag{17}$$

Eq. (13) is exact at the limit $N, T \to \infty$. For a randomly fluctuating time series, it is expected that distribution $\rho(\lambda)^E$ obtained by data analysis agrees with distribution $\rho(\lambda)^T$ calculated using Eqs. (13) to (17) for $\lambda \leq \lambda_{max}$. Therefore only a small number of eigen-values for $\lambda > \lambda_{max}$ have genuine correlation information.

In order to extract the genuine correlation, we rewrite correlation matrix C using eigen-value λ_α and the corresponding eigen-vector $|\alpha\rangle$ [13]. First we define the complex conjugate vector of eigen-vector $|\alpha\rangle$ by

$$\langle\alpha| = |\alpha^*\rangle^t. \tag{18}$$

For the real symmetric matrix, such as correlation matrix C, all elements of the eigen-vector $|\alpha\rangle$ are real, thus the complex conjugate means simply to transpose t.

Correlation matrix C then is rewritten as

$$C = \sum_{\alpha=0}^{N-1} \lambda_\alpha |\alpha\rangle\langle\alpha| \tag{19}$$

by multiplying Eq. (11) with transposed vector $\langle\alpha|$ from the left hand side and taking summation over α. Here, the property of the projection operator $|\alpha\rangle\langle\alpha|$

$$\sum_{\alpha=0}^{N-1} |\alpha\rangle\langle\alpha| = 1 \tag{20}$$

was used. As a result, correlation matrix C of Eq. (19) is divided in the following components:

$$C = C^t + C^r = \sum_{\alpha=0}^{N_t} \lambda_\alpha |\alpha\rangle\langle\alpha| + \sum_{\alpha=N_t+1}^{N-1} \lambda_\alpha |\alpha\rangle\langle\alpha|. \tag{21}$$

The first term C^t corresponds to the genuine correlation component ($\lambda > \lambda_{max}$). The second term C^r corresponds to the random component ($\lambda \leq \lambda_{max}$). The term $\lambda_0|0\rangle\langle 0|$ is interpreted as the change for a whole system, which is a comovement component of business cycles.

We introduce vector $|z(t)\rangle$, which consists of time series $z_i(t)(i = 1, \cdots, N)$. Then vector $|z(t)\rangle$ is expanded on the basis of eigen-vectors $|\alpha\rangle$ [13] :

$$|z(t)\rangle = \sum_{\alpha=0}^{N-1} a_\alpha(t)|\alpha\rangle. \tag{22}$$

Expansion coefficient $a_\alpha(t)$ is obtained using the orthogonality of the eigen-vectors:

$$a_\alpha(t) = \langle\alpha|z(t)\rangle. \tag{23}$$

The time series corresponding to the genuine correlation C^t is extracted by truncating the summation up to N_t in Eq.(22):

$$|z(t)\rangle = \sum_{\alpha=0}^{N_t} a_\alpha(t)|\alpha\rangle. \tag{24}$$

Business cycles fall into comovements and individual shocks. This classification is made using the random matrix theory. First, eigen-value λ_α and eigen-vector $|\alpha\rangle$

Table 1: Eigen-values and eigen-vectors

Parameter	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$	$\alpha = 5$
λ_α	2.767	1.033	0.793	0.613	0.444	0.346
α_1	-0.416	0.110	-0.271	0.849	-0.058	0.128
α_2	-0.472	-0.096	0.401	-0.051	-0.578	-0.517
α_3	-0.447	0.412	0.001	-0.166	0.667	-0.395
α_4	-0.256	-0.657	-0.643	-0.211	0.066	-0.196
α_5	-0.432	0.387	-0.271	-0.451	-0.329	0.526
α_6	-0.387	-0.475	0.526	-0.012	0.321	0.493

were obtained for correlation matrix C using Eq. (11) and the results are shown in Table 1. Then, $\lambda_{max} = 1.37743$ was estimated using Eqs. (14) to (17) for $N = 6$ and $T = 199$. The obtained results indicate that only the largest eigen mode ($\alpha = 0$) is meaningful and other eigen-modes are regarded as random noise.

Consequently, the comovement is reconstructed using Eq. (24) with $N_t = 0$. The individual shock is reconstructed with the remaining eigen-modes. It should be noted that each element of eigen-vector $|0\rangle$ has the same sign, which means that all of the countries have the same change in GDP. The obtained time series of the economic shocks are shown in Fig. 10.

We always observe significant individual shocks, which seem to occur randomly. A natural interpretation of the individual shocks is that "technological shocks". The present analysis demonstrates that fluctuations of average phases well explain business cycles, particularly recessions. As it is highly unlikely that all of the countries are subject to common negative technological shocks, the results obtained suggest that pure "technological shocks" cannot explain business cycles [13].

3.4 Coupled Limit-Cycle Oscillator Model

In this section, the mechanism of synchronization is discussed. The existence of a limit-cycle in business cycles was suggested in section 2.2. Based on this result, we develop a model of the international business cycles, based on the coupled limit-cycle oscillator model [14].

According to our previous paper [15], as changes in "kinetic energy" are equal to summed "power," a power balance equation,

$$\frac{d}{dt}\left[\frac{1}{2}I_i\dot{\theta}_i^2\right] = R_i - L_i - K_d\dot{\theta}_i^2 + \sum_{j=1}^{N} k_{ji}\sin\Delta\theta_{ji}. \tag{25}$$

is obtained. This model has the trade linkage structure depicted in Fig. 11. If the power is balanced, the oscillator rotates with constant speed $\dot{\theta}_i$.

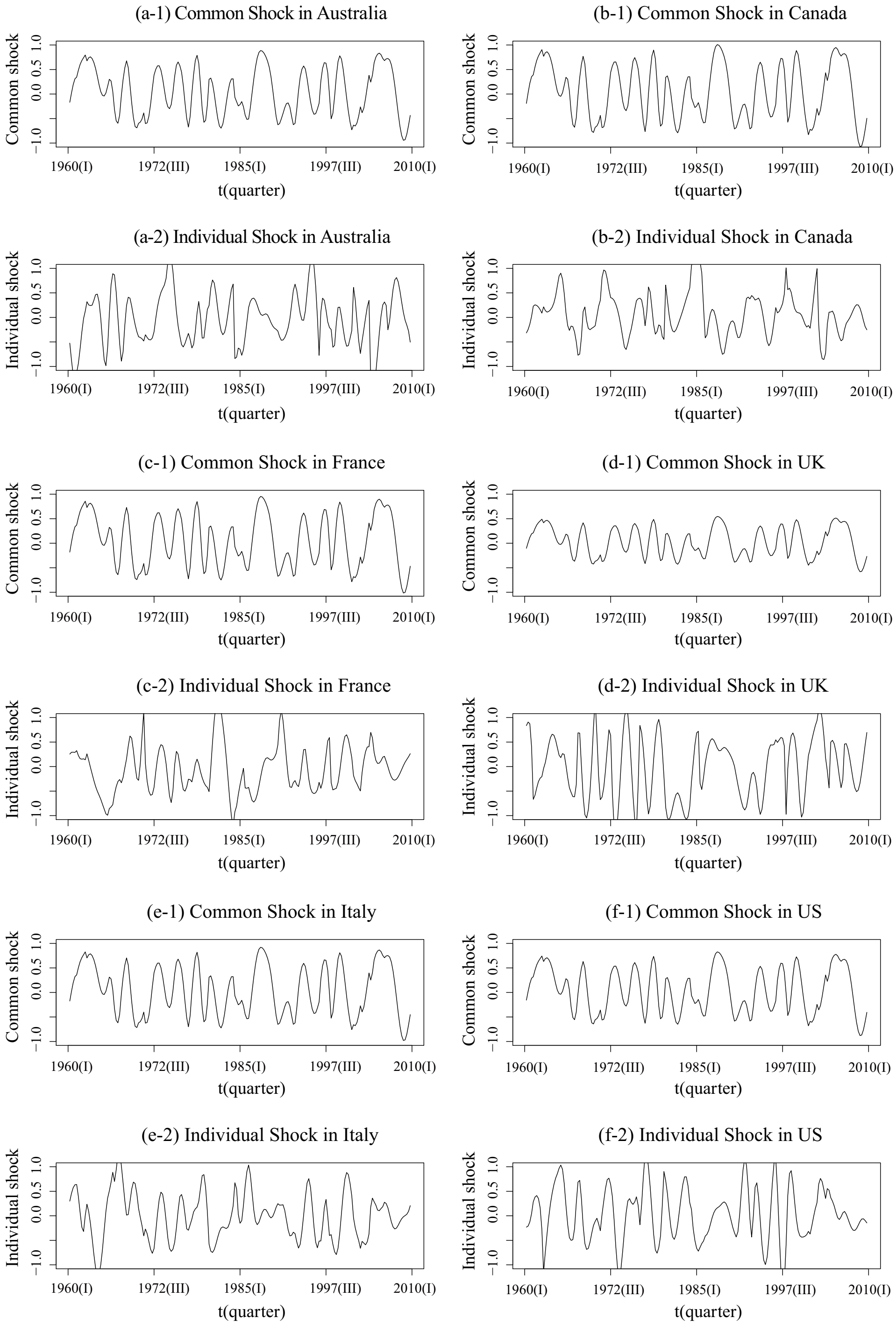

Figure 10: Common and Individual Shocks

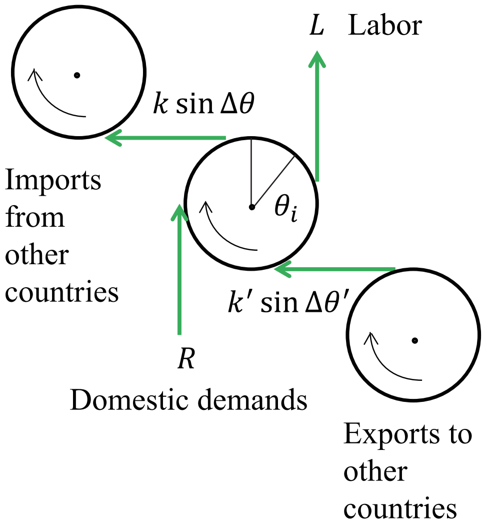

Figure 11: Trade Linkage Structure of the Coupled Limit-Cycle Oscillator Model

When the inertia term is small enough compared with the dissipation term ($\ddot{\theta}_i \ll \alpha_i \dot{\theta}_i$), the power balance equation leads us to obtain the Kuramoto oscillator, i.e. the coupled limit-cycle oscillator model [16],

$$K_d \dot{\theta}_i = R_i - L_i + \sum_{j=1}^{N} k_{ji} \sin \Delta\theta_{ji}. \tag{26}$$

Without the loss of generality, Eq. (26) is rewritten as,

$$\dot{\theta}_i = Q_i + \sum_{j=1}^{N} \kappa_{ji} \sin \Delta\theta_{ji}. \tag{27}$$

A theoretical study of this model has shown that the synchronization of oscillators is observed when interaction parameters κ_{ji} are greater than a certain threshold.

The parameter estimation then is explained. Using a discretized form of the model,

$$\theta_{i,t+1} = \beta_i \theta_{i,t} + Q_i + \sum_{j=1}^{N} \kappa_{ji} \sin \Delta\theta_{ji}, \tag{28}$$

the regression analysis was made to estimate the model parameters. The results are summarized in Appendix A.

Appendix A shows that the coupled limit-cycle oscillator model fits the phase time series of the GDP growth rate very well. The validity of the model implies that

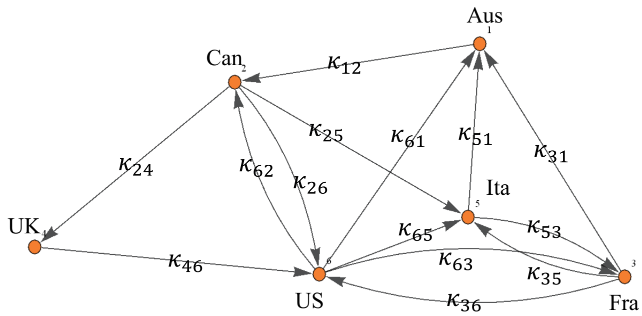

Figure 12: Network Structure of the Coupled Limit-Cycle Oscillator Model

the origin of the synchronization is the interaction due to international trade. The network structure of the model is shown in Fig. 12. Here the edge between i and j is shown, if the confidence interval of the interaction parameter κ_{ij} does not cross zero.

Furthermore, the mechanism of synchronization in the international business cycles is confirmed using simulations of the model as follows. In this simulation, a set of simultaneous differential equations,

$$\dot{\theta}_i = Q_i + \sum_{j=1}^{N} \kappa \sin \Delta\theta_{ji}, \tag{29}$$

is solved numerically with the assumed parameters, where the average and standard deviation are chosen to be the same with the regression estimations. For instance, parameter Q_i is uniform random variable over the interval $(0.35, 0.45)$, and initial values $\theta_i(0)$ are uniform random variable over the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. The interaction strength parameter κ was chosen in the range between 0.0 and 0.008. The results of the simulation are shown for different values ofκ in Fig. 13. In the case of $\kappa = 0.008$, synchronization is clearly reproduced. The simulations show that the threshold of the strength parameter κ is between 0.007 and 0.008. These results suggest that business cycles may be understood as dynamics of comovements described by the coupled limit-cycle oscillators being exposed to random individual shocks.

Finally, the relation between the size of trade and interaction strength is analyzed. Exports and imports relative to GDP are shown for Australia, France, the United Kingdom and the United States in Fig. 14. The ratios have increased for the last 20 years for all four countries. Trade data shows that the imports (exports) relative to GDP is high except for that of the United States. These figures show that the importance of international trade has increased and therefore the interaction between countries is expected to have been strong. In order to clarify the relation between the size of trade and interaction strength, the last 40 years were divided

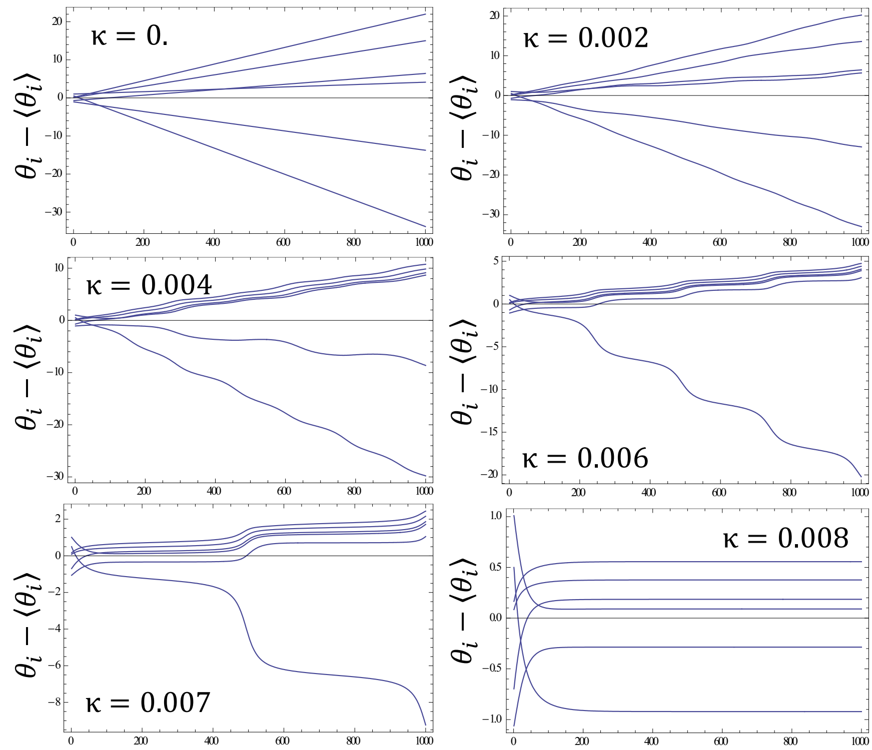

Figure 13: Synchronization and Interaction Strength

into four periods, i.e. period 1(1961-1980), period 2(1971-1990), period 3(1981-2000), and period4(1991-2010), and the parameter estimations using the regression analysis were made.

The overall strength indicators $S_i (i = 1, \cdots, N)$, defined by

$$S_i = \frac{1}{N} \sum_{j=1}^{N} \kappa_{ji}^2, \tag{30}$$

are shown in Fig. 15. The statistical error ϵ_i, defined by

$$\epsilon_i = \sqrt{\sum_{j=1}^{N} \left(\frac{2\kappa_{ji}}{N} \right)^2 \sigma_{ji}^2}, \tag{31}$$

is shown with indicator S_i. Here σ_{ji} is the standard error of κ_{ji}. The temporal change of the interaction strengths is shown for the six counties in Fig. 15, which depicts that the interaction strength indicators have increased for the last 40 years. These results clearly show that the interaction strength indicator became large in parallel with the increase in the size of exports and imports relative to GDP. Therefore, we conclude a significant part of the comovement comes from international trade.

4 Conclusions

We analyzed the quarterly GDP time series for Australia, Canada, France, Italy, the United Kingdom, and the United States from Q2 1960 to Q1 2010 to study the synchronization in the international business cycles. The followings results are obtained:

(i) The angular frequencies ω_i estimated using the Hilbert transform are almost identical for the six countries. This means that frequency entrainment is observed. Moreover, the indicator of phase locking $\sigma(t)$ shows that partial phase locking is observed for the analyzed countries. This is direct evidence of synchronization in the international business cycles.

(ii) A coupled limit-cycle oscillator model was developed in order to explain the mechanism of synchronization. Regression analysis showed that the model fits the phase time series of the GDP growth rate very well. The validity of the model implies that the origin of the synchronization is the interaction due to international trade.

(iii) Furthermore, we also showed that information from economic shocks is carried by phase time series $\theta_i(t)$. The comovement and individual shocks are separated using the random matrix theory. A natural interpretation of the individual shocks is that they are "technological shocks". The present analysis

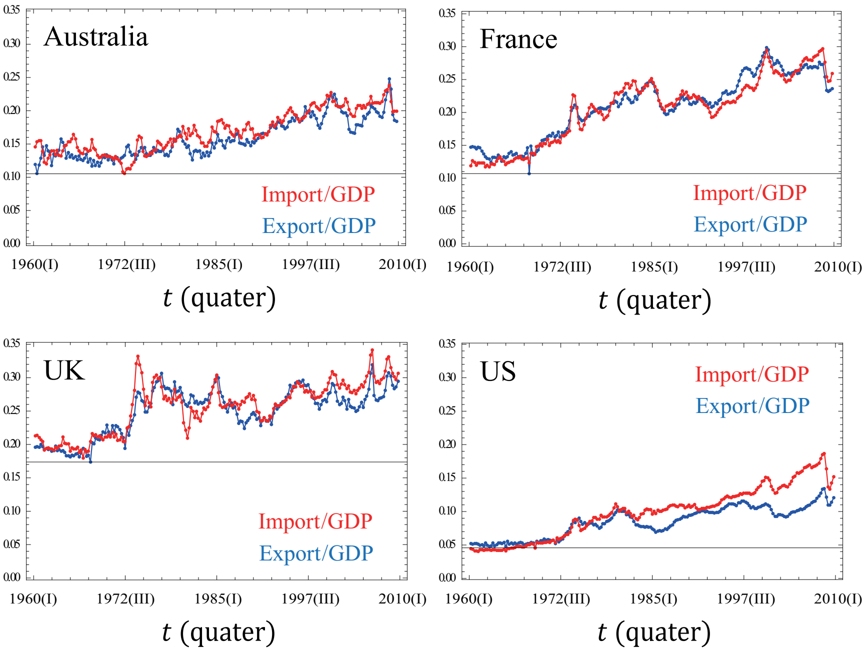

Figure 14: Temporal Changes of the Amount of International Exports and Imports relative to GDP

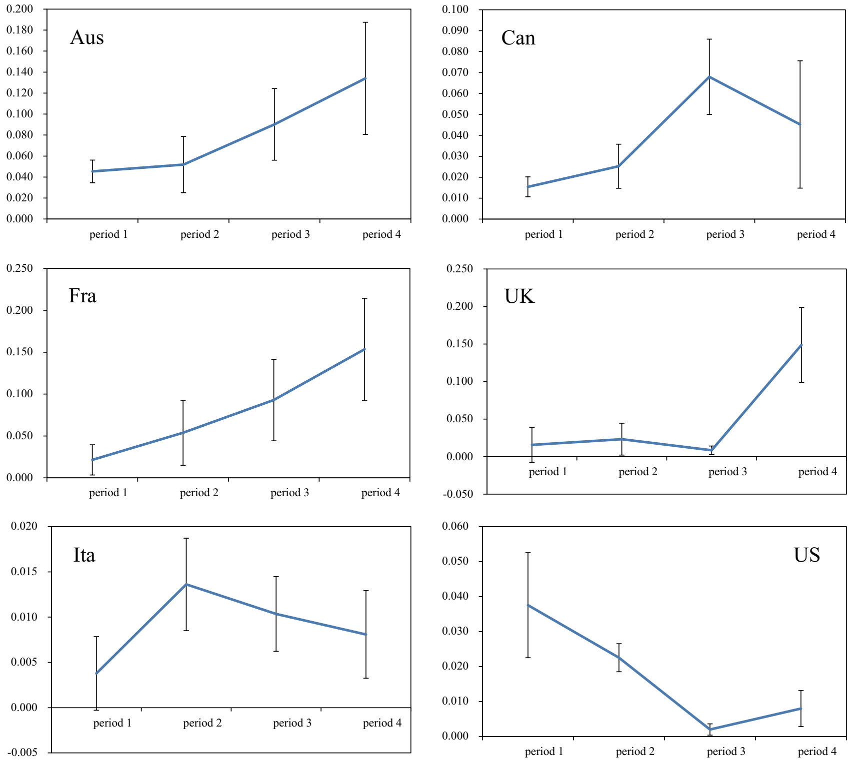

Figure 15: Temporal Changes of the Interaction Strengths

demonstrates that fluctuations of average phases well explain business cycles, particularly recessions. As it is highly unlikely that all of the countries are subject to common negative technological shocks, the results obtained suggest that pure "technological shocks" cannot explain business cycles.

(iv) Finally, the obtained results suggest that business cycles may be understood as dynamics of comovements described by the coupled limit-cycle oscillators exposed to random individual shocks. The interaction strength in the model became large in parallel with the increase in the size of exports and imports relative to GDP. Therefore, a significant part of comovements comes from international trade.

Acknowledgements

We thank H. Iyetomi, Y. Fujiwara, W. Souma, T. Watanabe, M. Fujita, M. Morikawa, and H. Ohashi for valuable discussion and comments. This work is supported in part by the Program for Promoting Methodological Innovation in Humanities and Social Sciences by Cross-Disciplinary Fusing of the Japan Society for the Promotion of Science.

Appendix A Parameter Estimation of Coupled Limit-Cycle Oscillator Model

Table 2: Australia

Parameter	Estimation	Std. Error	t value	Pr(> $\|t\|$)
β_1	0.996	0.001	590.090	< 2e-16
Q_1	0.412	0.076	5.427	2e-07
κ_{21}	-0.030	0.070	-0.433	0.665
κ_{31}	-0.174	0.063	-2.766	0.006
κ_{41}	0.051	0.047	1.080	0.281
κ_{51}	0.221	0.061	3.624	3e-04
κ_{61}	0.156	0.057	2.742	0.006

Multiple R-squared: 0.999, Adjusted R-squared: 0.999
F-statistic: 6.311e+04 on 6 and 191 DF, p-value: < 2.2e-16

Table 3: Canada

Parameter	Estimation	Std. Error	t value	Pr(> $\|t\|$)
β_2	0.996	0.001	814.971	< 2e-16
Q_2	0.404	0.050	8.023	1e-13
κ_{12}	0.100	0.044	2.266	0.024
κ_{32}	-0.025	0.052	-0.478	0.633
κ_{42}	0.033	0.038	0.854	0.394
κ_{52}	0.014	0.046	0.309	0.757
κ_{62}	0.300	0.045	6.549	5e-10

Multiple R-squared: 0.999, Adjusted R-squared: 0.999
F-statistic: 1.265e+05 on 6 and 191 DF, p-value: < 2.2e-16

Table 4: France

Parameter	Estimation	Std. Error	t value	Pr(> $\|t\|$)
β_3	0.995	0.002	445.277	< 2e-16
Q_3	0.397	0.076	5.206	5e-07
κ_{13}	0.007	0.059	0.126	0.900
κ_{23}	0.073	0.073	0.990	0.323
κ_{43}	0.041	0.059	0.690	0.491
κ_{53}	-0.145	0.072	-1.996	0.047
κ_{63}	0.154	0.064	2.394	0.017

Multiple R-squared: 0.999, Adjusted R-squared: 0.999
F-statistic: 4.851e+04 on 6 and 191 DF, p-value: < 2.2e-16

Table 5: UK

Parameter	Estimation	Std. Error	t value	Pr(> $\|t\|$)
β_4	0.996	0.002	459.586	< 2e-16
Q_4	0.427	0.077	5.545	9e-08
κ_{14}	-0.041	0.060	-0.683	0.495
κ_{24}	-0.188	0.073	-2.550	0.011
κ_{34}	-0.053	0.066	-0.811	0.418
κ_{54}	0.086	0.073	1.168	0.244
κ_{64}	0.114	0.067	1.681	0.094

Multiple R-squared: 0.999, Adjusted R-squared: 0.999
F-statistic: 4.494e+04 on 6 and 191 DF, p-value: < 2.2e-16

Table 6: Italy

Parameter	Estimation	Std. Error	t value	Pr(> $\|t\|$)
β_5	0.999	0.001	997.140	< 2e-16
Q_5	0.330	0.034	9.658	< 2e-16
κ_{15}	0.011	0.026	0.429	0.668
κ_{25}	-0.095	0.032	-2.960	0.003
κ_{35}	0.112	0.035	3.221	0.001
κ_{45}	-0.034	0.027	-1.288	0.199
κ_{65}	0.094	0.028	3.268	0.001

Multiple R-squared: 0.999, Adjusted R-squared: 0.999
F-statistic: 2.222e+05 on 6 and 191 DF, p-value: < 2.2e-16

Table 7: USA

Parameter	Estimation	Std. Error	t value	Pr(> $\|t\|$)
β_6	0.998	8e-01	1236.896	< 2e-16
Q_6	0.450	0.039	11.511	< 2e-16
κ_{16}	-0.002	0.033	-0.077	0.938
κ_{26}	-0.159	0.036	-4.323	2e-05
κ_{36}	-0.104	0.035	-2.983	0.003
κ_{46}	-0.105	0.032	-3.256	0.001
κ_{56}	0.056	0.032	1.715	0.087

Multiple R-squared: 0.999, Adjusted R-squared: 0.999
F-statistic: 2.718e+05 on 6 and 191 DF, p-value: < 2.2e-16

References

[1] G. Von Haberler, "Prosperity and Depression: A Theoretical Analysis of Cyclical Movements," League of Nations (Geneva), 1937.

[2] A. F. Burns and W. C. Mitchell., "Measuring Business Cycles," National Bureau of Economic Research (Studies in business cycles; 2), 1964.

[3] C.W.J. Granger and M. Hatanaka, "Spectral Analysis of Economic Time Series," Princeton: Princeton University Press,1964.

[4] C. Huygens, "Horologium oscillatorium: 1673," Michigan: Dawson, 1966.

[5] P. R. Krugman, "The Self-Organizing Economy," Cambridge, Mass., and Oxford: Blackwell Publishers, 1996.

[6] J. H. Stock and M. W. Watson, "Understanding Changes in International Business Cycle Dynamics," *Journal of the European Economic Association*, 3 (5), pp.968-1006, 2005.

[7] A. T. Foerster, P. G. Sarte, and M. W. Watson, "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production," *Journal of Political Economy*, 199 (1), pp.1-38, 2011.

[8] D. Gabor, "Theory of communication. Part 1: The analysis of information," *J. Inst. Elect. Eng.*, 93, pp.429-441, 1946.

[9] M. L. Metha, "Random Matrices," Academic Press (Boston), 1991.

[10] L. Laloux, P. Cizeau, J.-P. Bouchaud, M. .Potters, *Phys. Rev. Lett.* 83, 1467 (1999).

[11] V. Pleroux, P. Gopikrishnam, B. Rosenow, L. A. N. Amaral, and H. E. Stanley, *Phys. Rev. Lett.* 83, 1471 (1999).

[12] V. Pleroux, P. Gopikrishnam, B. Rosenow, L. A. N. Amaral, T. Guhr, and H. E. Stanley, *Phys. Rev. E.* 65, 066126 (2002).

[13] H. Iyetomi, Y. Nakayama, H. Yoshikawa, H. Aoyama, Y. Fujiwara, Y. Ikeda, and W. Souma, "What Causes Business Cycles? Analysis of the Japanese Industrial Production Data," *Journal of Japanese and International Economics*, Volume 25, Issue 3, pp.246-272, 2011.

[14] Y. Kuramoto, "Chemical Oscillations, Waves, and Turbulence," Springer-Verlag Berlin Heidelberg, 1984.

[15] Y. Ikeda, H. Aoyama, Y. Fujiwara, H. Iyetomi, K. Ogimoto, W. Souma, and H. Yoshikawa, "Coupled Oscillator Model of the Business Cycle with Fluctuating Goods Markets," *Progress of Theoretical Physics Supplement*, No.194, pp.111-121, June 2012.

[16] Y. Ikeda, H. Aoyama, H. Iyetomi, H. Yoshikawa, "Direct Evidence for Synchronization in Japanese Business Cycle," arXiv:1305.2263v1 [q-fin.ST] 10 May 2013.