DebtRank Analysis of the Japanese Credit Network

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Abstract

We present an analysis of the lending/borrowing relationship between Japanese banks and Japanese firms, which form a bipartite credit network. We introduce distress to some initial node(s) (banks or firms) and allow it to propagate and contaminate other nodes in this network according to the relative exposure. First, by choosing the initial node to be a bank and taking the weighted average of the resulting distress distribution, with the weight proportional to the size (total assets) of each node, we identify the bank’s importance to the whole network at the time of crisis. This leads to a nonlinear relationship between the importance and the size of the bank, which implies that mergers with the same-sized partner would result the most in the increase in importance. Second, by introducing the initial distress to firms in certain industrial sector(s), we evaluate the vulnerability of banks and firms in other sectors due to the distress in the initial sectors.

Keywords: Bank lending; Economic crisis; Bank monitoring

JEL classification: G01, G21
I. DebtRank on a generic network

Consider a network with nodes \( i = 1, 2, 3, \cdots, N \) with links that are directional and weighted by \( w_{ji} \in [0, 1] \), which represent weight of propagation of financial distress from the node \( j \) to the node \( i \).

DebtRank (Battiston et al., 2012) quantifies the amount of distress generated and distributed to the whole network from a particular node or a set of nodes. The higher the total network-distress, the higher the DebtRank of the initial node(s). The way distress is distributed to the network is calculated in several (finite) time-steps. At each time-step \( t (= 0, 1, 2, 3, \cdots) \), two state-variables are assigned to each node \( i \):

1. \( h_i(t) \in [0, 1] \), the amount of distress of the node \( i \) at time \( t \).
2. \( s_i(t) \in \{U, D, I\} \), the node \( i \) is one of the states, ‘Undistressed’, ‘Distressed’, or ‘Inactive’, respectively at time \( t \). This variable is introduced to avoid infinite repetition of propagation (circulation) of the distress in loops, as we will see below.

In order to evaluate the DebtRank of the nodes in the group \( A \), we first assign

\[
h_i(0) = \begin{cases} 
1 & \text{if } i \in A, \\
0 & \text{otherwise}, 
\end{cases} \quad (1)
\]

\[
s_i(0) = \begin{cases} 
D & \text{if } i \in A, \\
U & \text{otherwise}, 
\end{cases} \quad (2)
\]

Then at each step we propagate the distress according to the following rules.

\[
h_i(t) = \min \left[ 1, h_i(t-1) + \sum_{j} w_{ji} h_j(t-1) \right] \quad (3)
\]

\[
s_i(t) = \begin{cases} 
D & \text{if } h_i(t) > 0 \land s_i(t-1) \neq I, \\
I & \text{otherwise}. 
\end{cases} \quad (4)
\]

The quantity \( w_{ij} \) governs the ratio of the distress propagated from the node \( i \) to the node \( j \), and we may call it the ‘propagation matrix element”. By these rules, a node \( i \) who started as a D-state (at \( t = 0 \)) becomes an I-state at \( t = 1 \) and stops further propagation of distress. Other nodes who started as a U-state becomes a D-state when distress reaches it, and once it contaminate other site with distress at the next time-step it becomes an I-state and stops further propagation of the distress. Note that an I-state can still receive distress.

Once every node becomes an I-state (at time-step \( t = T \)) and propagation of distress stops, we have a list of distress \( h_i(T) \) from which we may define various
weighted averages, which are called **debtranks**. Battiston et al. (2012) defined an weighted average of all the nodes with weights proportional to the size of the nodes. In this paper, we have two layers of nodes, for which we define two weighted averages, as we will elaborate in the following section.

II. Bank-Firm Credit Network

The database we analyse is an annual list of bank-loans (both long-term and short-term) to firms in Japan, which form a weighted bipartite network (Fujiwara et al., 2009).²

It covers all banks (including ‘saving and loans’ type of monetary institution, called “regional banks”) and large firms, most of which are listed. Their total numbers for years 1980 to 2011 are plotted in Fig.1.³ A network visualization is given in Fig.II, where we see that city banks (red squares) are in the center of the upper layer, meaning that they have many links (loans) to firms in the lower layer, and therefore are playing major roles, while regional banks (blue and green) are around the peripheral, indication that they play peripheral roles in this system, although they are numerous in numbers.

We denote banks and firms by Greek letters $\beta (\mu = 1, 2, \cdots B)$ and Latin letters $f (f = 1, \cdots, F)$ respectively, and $B$ is the number of banks, and $F$ is that of firms. An edge between a bank $\beta$ and a firm $f$ is defined to be present if there is a credit relationship between them.

We note that there is no inter-bank edges or inter-firm edges in our system. Technically, this is simply due to the fact that they are not available to us currently.

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²Partial and preliminary results of the following were presented at Aoyama (2013a,b); Battiston (2013).

³In addition, the MST of banks, which shows strong regional structure, and each sector’s annual behavior are shown in the appendices.

⁴The notation differs from Fujiwara et al. (2009), so that so that the name (bank or firm), the index ($\beta$ and $f$) and the total numbers ($B$ and $F$) match with each other.
One, however, might argue that they are not relevant for our current purpose, which is to examine banks’ importance and vulnerability at the time of a crisis: Any distress or failure of a bank might affect to the whole economic system, as other banks tries to absorb its damage by government intervention, not by any pre-crisis inter-bank relationship. Firms may go thorough chain-bankruptcy due to bankruptcy of other firms with whom they trade, for which we need information of inter-firm trading network. But its effect to the whole system may go through their strongest ties to the banks, as they fail to repay the loans. For these reasons, we believe that the current analysis with bank-firm bipartite network data would give us a good clue to the importance and vulnerability of bank(s).

The amount $C_{\beta f}$ associated with the edge is the amount of the credit (total lending by the bank $\beta$ to the firm $f$), as illustrated in Fig.3. The propagation matrix element $w_{ji}$ is defined to the relative exposure;

$$w_{f\beta} := \frac{C_{\beta f}}{C_{\beta}}, \quad \tag{5}$$

$$w_{j\beta f} := \frac{C_{\beta f}}{C_{f}}. \quad \tag{6}$$
The quantity $C_{\beta f}$ is the sum of the long-term loan and the short-term loan of that the year in discussion from the bank $\beta$ to the firm $f$.

$$C_{\beta} := \sum_{f} C_{\beta f}, \quad \text{(7)}$$

and likewise

$$C_{f} := \sum_{\beta} C_{\beta f}. \quad \text{(8)}$$

Figure 3. The quantity $C_{\beta f}$ is the sum of the long-term loan and the short-term loan of that the year in discussion from the bank $\beta$ to the firm $f$.

Figure 4. CDF (cumulative distribution function) of $C_{\beta}$ (left) and $C_{f}$ (right) from the year 1980 to 2012.

where $C_{\beta}$ is the total amount of lending by bank $\beta$;

The CDFs (Cumulative Distribution Functions) of $C_{\beta}$ and $C_{f}$ are plotted in Fig.II from 1980 to 2012.

The propagation matrix element (5) reflects the fact that once the firm $f$ is in distress it affects the banks $\beta$, from whom the firm $f$ has borrowed the amount $C_{\beta f}$, through delayed payment of the interest payments, even a total failure to repay the borrowed money, etc., etc. We model this by using relative exposure of the bank to the firm and not the absolute amount, because if the bank is lending a lot more to other firms, effect of the firm $f$ would be small, and vise versa. The latter weight (6) rises from the fact that once a bank $\beta$ is in distress, it affects the firms that are borrowing through rising of the interest rate, request for further security, etc., which are again relative exposer if the firm to the bank. Note that these are identical to $A$ and $B$ in Fujiwara et al. (2009), respectively.
As stated in the previous section, after all the distress propagation is over at time-step \( t = T \), we define two weighted averages of the distress: The debtrank on the bank layer;

\[
D_{A}^{(\text{banks})} = \sum_{\beta=1, \beta \in A}^B \hat{a}_\beta h_\beta (T), \quad \hat{a}_\beta := \frac{a_\beta}{\sum_{\beta'=1, \beta' \notin A}^B a_{\beta'}},
\]

where \( a_\beta \) is the total asset of the bank \( \beta \), so that the larger the node is the larger its distress is counted in. Note that we exclude the node in the initial set \( A \) in the sums, so that the resulting debtrank is not a simply reflection of the initial node(s): without this, a large initial node contributes (initial) \( h = 1 \) with large weight, thus resulting large debtrank. In other words, by excluding the initial node(s), our debtrank is a measure how the initial node(s) affect other nodes in the network.

Similarly, the debtrank on the firm layer;

\[
D_{A}^{(\text{firms})} = \sum_{f=1}^F \hat{a}_f h_f (T), \quad \hat{a}_f := \frac{a_f}{\sum_{f'=1}^B a_{f'}},
\]

where \( a_f \) the total asset of the firm \( f \). This debtrank on the firm layer \( D_{A}^{(\text{firms})} \) is a measure of the distress caused on firms.

By using these two debtranks, \( D_{A}^{(\text{banks})} \) and \( D_{A}^{(\text{firms})} \), we shall quantify importance and vulnerability of the node(s) \( A \).

### III. “Too big to fail?” and other questions.

We first choose the set \( A = \{ \beta_0 \} \) to evaluate the bank \( \beta_0 \) ’s importance in the system. The result for the year 2010 is given in Fig. 5 in linear-scale (left) and in log-scale.
The dashed line in the latter plot is the best-fit power-law,
\[ D_{(\text{firms})} = 1.76D_{(\text{banks})}^{1.13}, \] (11)
which hold as a good relationship as an average and any deviation from this is a measure of their characteristics in lending practice: Large banks such as Tokyo Mitsubishi UFJ and Mizuho Corporate shows that their effect on firms are larger, while Mizuho and Risona have more effect on other firms. Trust banks in general have higher effect on firms than the average, which is a natural consequence of the fact that their role is to support firms in financing and management services (Trust Companies Association of Japan, 2013).

This plot and the average behavior above have several implications, which we shall elaborate on below.

Fig.(6) is the distribution of the banks on the plane of (Asset, The total debtrank \((D_{(\text{banks})} + D_{(\text{firms})})\)). We find here a general trend that the larger the bank is the larger the total debtrank, with the best-fit
\[ D_{(\text{banks})} + D_{(\text{firms})} = 6.55 \times 10^{-19} S^{1.50}, \] (12)
where \(S\) is the size (total asset) of the bank.

**Too big to fail?** Nonlinearity, the power behavior in Eq.(12) evidently shows the importance of the size of the bank. This, no matter how trivial it sounds,
is relevant in the context of debtrank, as our debtrank excludes the bank in question (see Eq.(9)) and measures it importance to other banks and firms. Therefore, we obtain an independent and objective conclusion that the big banks are important in average. On the other hand, if one looks at the deviations from this average behavior, we find that the above statement is not always true. The biggest bank, Tokyo Mitsubishi UFJ has total asset twice as large as the Mizuho Corporate but has about the same level of debtrank. Similarly Mizuho’s total asset is about 2.7 times as large as that of the Risona and yet they have same level of debtrank, and so forth. Most of these deviations from the average behavior comes from difference in their lending practice: In general trust banks, as noted before, stressed their role in lending to firms and this explains their high debtranks. Shikin Central Bank’s case is another case: it plays a role of the central bank for all the trust banks, and its importance is underrated from this analysis of lending to firms.

**Nonlinearity and merger**  The fact that the exponent is larger than one (≈ 1.50 in Eq.(12)) is a significant result: it means that if the bank becomes, say, twice as big, its total debtrank becomes 2.82 times as before. Therefore, it means more than simply stating that “big banks are important”, but more like “big banks are far more important than small banks.” This provides a strong motivation for merger: Let us think of bank mergers in a more practical situation. Imagine two banks of size $S_1$ and $S_2$. Since

$$S_1^\alpha + S_2^\alpha < (S_1 + S_2)^\alpha,$$

for $\alpha > 1, S_1 \neq 0, S_1 \neq 0$, their merger will result in the total debtrank that is larger than the sum of their debtrank. In fact, their ratio is a function of $S_1/S_2$:

$$\frac{(S_1 + S_2)^\alpha}{S_1^\alpha + S_2^\alpha} = R\left(\frac{S_1}{S_2}\right),$$

which has a maximum at $S_1/S_2 = 1$ regardless of the value of the exponent $\alpha$ as long as it is greater than 1 (Fig.7), with the peak value $R(1) = 2^{\alpha-1}$. Therefore, we conclude that by merging with the equal-sized partner, they achieve maximum increase in their importance.\(^5\)

We note that above is a general discussion of merger and individual cases differ form this because of several reasons, some of which are; (1) the deviation of the agents from the average curve, (2) merger is associated with separation of divisions from either agents and asset management/reduction, as they tries to cope with difficult transition, and (3) the above discussion ignores the changes in the lending structure by merger: more specifically, the propagation matrix elements (5) and (6) are not homogeneous in the amount of the lending $C_{\beta f}$.

\(^5\)It is curious to note that if $\alpha < 1$, the total debtrank is always less than the sum of the total debtranks before the merger, whose ratio is minimum at $S_1 = S_2$.  

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Figure 7. Plot of the importance-ratio function $R(x) = (x+1)^\alpha/(x^\alpha + 1)$ in Eq.(14) for $\alpha = 1.50$.

For example, Sanwa Bank and Tokai Bank merged to form the UFJ bank in 2002. Before the merger, their debtranks are 0.23 and 0.14, respectively and after the merger, it is 0.33, slightly less than the simple some of the debtranks before the merger, in contradiction to the general theory. Their total asset, however, were reduced by 20%, which disqualifies the direct application of the general theory.

IV. Vulnerability

The distress $h_{f,\beta}$ can be used as an measure of venerability of nodes in this network.

Let us put firms in certain industrial sectors in distress, by choosing them to be in the initial distress set $A$ in Eqs.(1) and (2), and all the other firms and banks $\not\in A$. Then, the resulting distress $h_{\beta}$ (after all the propagation) is the measure of vulnerability of the bank $\beta$ to the failure or distress in those sectors.

First, we choose $A$ to consist of all the firms in all of the manufacturing sector, and measure the vulnerability of banks. Then we may do the same for all of the non-manufacturing (service) sectors. Fig.8 is the plot of the resulting vulnerabilities, where the dashed line is the best-fit linear function, $y = 1.18 - 0.60x$ with $x$ and $y$ being the horizontal and the vertical coordinates, respectively. We observe here that the city banks are well balanced, while regional and 2nd regional banks distribute widely, which is in agreement with the fact city banks are large and lend to a wide spectrum of firms, while the (2nd) regional banks tend to be small and their lending may be limited to a small set of firms. On the other hand, we see that this method of studying vulnerability is a powerful one in identifying small firms with limited lending practice that makes them quite vulnerable to various systemic crisis.

Another useful study is the vulnerability of banks to distress in the automobile sector, and the same from the construction sector, since they are most affected by external shocks, as was true at the time of Lehman shock. The results are shown in Fig.9 together with their total assets.
Doing this kind of analysis for all 33 sectors (see Table IV), we find the resulting distress to all 159 banks shown in Fig. 10. In the top matrix view, we observe bright (white to red) column, which is the sector No. 23, “Credit & Leasing”, which is followed by No. 22, “Retail Trade” and then by No. 21 “Wholesale Trade”, which we find are important sectors for stability of monetary institutions.

Let us look at the vulnerability of a sector to another sectors, which we measure by mean $h_f$ for the firms in the former sector, caused by propagation of distress of the firms in the latter sector. Result is shown in Fig. 11. Here again we observe the bright column apparent in the matrix view (left), which is the sector No. 23, “Credit & Leasing”, in agreement with the above observation.
Figure 9. Vulnerability of banks to distress in automobile sector (left) and the construction sector (right).
Figure 10. Vulnerability (Distress) of 159 banks caused by each of 33 sectors in 2010, in matrix view (top) and 3D view (bottom).

Figure 11. Vulnerability (Distress) of sectors to other sectors, in matrix view (left) and 3D view (right).
<table>
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<tr>
<th>No.</th>
<th>Nikkei code</th>
<th>Sector</th>
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<tbody>
<tr>
<td>1</td>
<td>101</td>
<td>Foods</td>
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<tr>
<td>2</td>
<td>103</td>
<td>Textile Products</td>
</tr>
<tr>
<td>3</td>
<td>105</td>
<td>Pulp &amp; Paper</td>
</tr>
<tr>
<td>4</td>
<td>107</td>
<td>Chemicals</td>
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<tr>
<td>5</td>
<td>109</td>
<td>Drugs</td>
</tr>
<tr>
<td>6</td>
<td>111</td>
<td>Petroleum</td>
</tr>
<tr>
<td>7</td>
<td>113</td>
<td>Rubber Products</td>
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<tr>
<td>8</td>
<td>115</td>
<td>Stone, Clay &amp; Glass Products</td>
</tr>
<tr>
<td>9</td>
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<td>Iron &amp; Steel</td>
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<td>119</td>
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<td>241</td>
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<tr>
<td>33</td>
<td>271</td>
<td>Other Services</td>
</tr>
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</table>

Table 1. All 33 sectors specified in the Nikkei database. Code on 100’s (No.1-17) are manufacturing sectors, 200’s (No.18-33) are non-manufacturing sectors.
V. Summary and Implications

We have shown that debtrank approach is a powerful one that provides us with a measure of importance of the nodes as well as vulnerability of the nodes at times of crisis.

Two debtranks are defined, one for the bank-layer and the other for the firm-layer and found that they are almost equal with slight non-linearity, as in Eq.(11). Some of those, the total debtrank, is a good measure of their importance in the network. In average, it has a good correletion with the bank’s size (total asset) as in Eq.(12), whose exponent 1.50 implies that merger with the same-sized partner is the optimal solution in the increase of the total debtrank. 6

Vulnerability is measured by how much the node(s) in question receives distress from particular set of node(s). This has a wide range of applications for investigating the weak spots to particular type of crisis. In this paper, we have identified banks that are quite vulnerable to crisis in the automobile sector. We have found that the distress in the “Credit & Leasing sector” affects all sectors most, which implies that it is fairly important to keep them in good financial standing.

In the current research we have mainly reported the results of the analysis of 2010. The data we have, however, covers 1980 to 2012 and therefore comparing results of each year would be important, which we wish to report in future.

There are several missing links in our consideration, which are interbank (bank-bank) and interfirm (firm-firm) interactions. The latter may be constructed out of trading information available from databank firms in Japan, while the latter is available to the current authors presently.

Another important improvement of the DebtRank approach in sight is some way to incorporate the constraint on capital-to-asset ratio for banks by the Basel Capital Accords. This may act as a threshold to the distress variable, so that once the ratio is below the 8% mark, distress variable may be enhanced. We may look into this modification of the current method in near future.

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6This is somewhat analogous to the supermodularity concept in game theory. The exact relation between them, however, will be investigated in near future. The authors would like to thank Prof. M. Fujita, the Chief Research Officer at RIETI, for pointing this out to us.
Appendix A. Similarity of lending and geographical regions

Lending patterns of financial institutions have similarity due to geographical regions. It is obvious that regional banks are often lending to a limited but overlapping set of firms in the same local regions, and also that city and mega banks provide credit to many and overlapping set of firms. As we have shown, some results of the DebtRank analysis can be interpreted in terms of the similarity of lending, which we elaborate in this appendix.

Let us define a distance between a pair of banks based on their lending patterns. A lending pattern of a bank $\beta$ is a vector $x_\beta$ of dimension $F$, the number of firms. Each component is given by

$$(x_\beta)_f = \begin{cases} 1 & \text{if } \beta \text{ lends to } f \\ 0 & \text{otherwise} \end{cases},$$

for $f = 1, \cdots, F$, namely 1 or 0 according to the presence or absence of the credit relation between bank $\beta$ and firm $f$. Alternatively one could use the information of lending weight, but it is sufficient to use the information of 0/1 for our purpose. Then we normalize the vector so that $\|x_\beta\| = 1$, and define the distance between $\beta$ and $\alpha$ by

$$d(\beta, \alpha) = \|x_\beta - x_\alpha\| = \sqrt{2(1 - x_\beta \cdot x_\alpha)}.$$  

We employ the well-known method of minimum spanning tree (MST) as a compact representation for depicting the entire map of similarities among banks. An MST is an undirected and tree graph of nodes $\beta = 1, \cdots, B$ and $(B - 1)$ edges. Assume that the edge for a pair of nodes $(\beta, \alpha)$ has a weight equal to $d(\beta, \alpha)$. One can construct such a tree in an arbitrary way, and the MST is the one among them such that the total sum of weights of resulting edges in the tree is minimum. By this construction, a pair of nodes whose distance is small is more likely to be located with a smaller shortest-path in the tree. Therefore the banks are likely located to be close in the MST.

Fig. 12 is the result of the data in the year 2010. We can identify geographical regions in the MST as sets of banks having similar patterns specific to those regions, as shown by circles with colors corresponding to eight regions in Japan. Also, in the bottom and left portion, a set of city mega banks is present implying that they provide credit to many and overlapping firms.
Figure 12. Minimum spanning tree for the nodes of banks and the edges with weights of distances between banks. Distance of a pair of nodes is smaller if the banks have similar lending to firms. The node size is proportional to the logarithm of the bank’s asset. Circles with colors are identified as geographical regions and city mega banks. (Upper-left) Japan and eight geographical regions; Hokkaido (red), Tohoku (yellow), Kanto including Tokyo (green), Chubu (cyan), Kansai including Osaka and Kyoto (blue), Chugoku (orange), Shikoku (purple), Kyushu including Okinawa (gray).
Appendix B. Quantifying shocks in industrial sectors

We can quantify a shock in a particular industrial sector by using the financial states of firms within the sector. Denote an industrial sector by $g$, and let the sales of a firm $f$ in the sector at year $t$ be $S_i(t)$. Annual growth-rate is defined by

$$R_i(t) = \frac{S_i(t)}{S_i(t-1)},$$  \hspace{1cm} (B.1)

where $S_i(t-1)$ is the sales in the previous year $t-1$. We measure the extent of idiosyncratic shock in a sector by the average of logarithmic growth-rate:

$$r_g(t) = \frac{1}{n_g} \sum_{i} \ln R_i(t),$$  \hspace{1cm} (B.2)

where the summation is taken over all the firms in the sector, and $n_g$ is the number of them. In other words, $r(t)$ is the logarithm of geometric mean for the growth-rates. Geometric mean is suited for our purpose, because it is a robust measure that can capture the “average” shocks in the industrial sector.

We used our dataset focusing on the firms and their financial statements, and calculated $r_g(t)$ for sectors $g$ with sufficient number of firms, $n_g$, from 1981 to 2012. Fig. 13 shows the result. As observed from the plots for different sectors, we can identify similarity and difference between the shocks in sectors. For example, the Lehman crisis in 2008 and subsequent shocks caused considerable recession in manufacturing sectors such as metals, machinery and electronics, and also in non-manufacturing sectors such as wholesale trade and retail trade, in corresponding years of 2009 and subsequent years. On the other hand, other sectors including foods, chemicals and construction have less extent of shocks during the years.
Figure 13. Each plot shows the shocks in different industrial sectors from 1981 to 2012, which are quantified by average logarithm of annual growth-rates for firms in the sector. Sectors (from top to bottom): foods, chemicals, iron & steel and metals, machinery, electronics, motor vehicles & auto parts, construction, wholesale and retail trade.
References


