Trade Costs, Wage Difference, and Endogenous Growth

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Abstract
In this paper, we develop an endogenous growth model with two countries in which the international trade of differentiated goods requires trade costs and equilibrium wages in the two countries are different. With this model, we show that both wage differences and market size have important effects on the location of manufacturing firms and the innovation sector as well as on economic growth.

First, when trade costs are high, the share of manufacturing firms in the large country increases with a decline in trade costs because of market size. However, the share of firms then decreases with a decline in trade costs when trade costs are low because of wage differences. Finally, all firms agglomerate in the small country, since production costs there are low. In this process, the innovation sector shifts its location from the large-market, high-wage country to the small-market, low-wage country.

In this globalization process, growth rates first increase, then decrease, and finally increase with the reduction of trade costs. These results explain the process of the initial high growth of developed countries, location shift of manufacturing firms, and innovation sector from developed to developing countries, which has been observed in recent years.

Keywords: Market size, Wage differential, Growth rates, Innovation sector.

JEL classification: F0, O31.

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1 Introduction

A feature of recent world economy is a decline in trade costs. Trade costs include tariffs, informational, cultural, industrial barriers, and transportation costs. Bordo et al. (1999) reported that average tariffs fell from around 20% in 1950 to under 5% in recent years, and nontariff barriers, such as quotas and exchange controls, were largely removed. In addition, recent improvements in transportation technologies, such as aviation, containerization, and bulk shipping, have lowered transportation costs. These movements derive a decline in trade costs.

With these reductions in trade costs, manufacturing activity has recently been gradually shifting from developed to developing countries. Gao (2007) reported that US manufacturing employment steadily has declined at an annual rate of 0.4% in the last 35 years. In Japan, the percentage of manufacturing labor to total labor decreased from 36.6% in 1973 to 24.8% in 2010. Manufacturing activities shifted from Japan to East Asian countries such as Korea, Chinese Taipei, Thailand, and China.

With this move of the manufacturing sector, recently, R&D activities have flourished in these countries. OECD (2010) reported that the Chinese R&D expenditure to total OECD R&D expenditure was 5% in 2001 and 13.1% in 2008. This report noted that the recent growth rates of R&D expenditure of the BRICs countries (Brazil, Russia, India, and China) are rapid and important. On the other hand, OECD (2010) showed that the growth rates of R&D expenditure in OECD countries have decreased in recent years. Therefore, R&D activities following the manufacturing sector started to shift from developed to developing countries.

Before the move of the manufacturing sector from Japan to East Asia, agglomeration of manufacturing activities supported the rapid growth of Japan. The share of manufacturing labor to the total labor in Japan increased from 24.3% in 1950 to 36.6% in 1973. Therefore, agglomeration of manufacturing activities progressed from 1950 to 1970, and the hollowing out of manufacturing activities was observed from 1970 to 2010 with the monotonic reduction of trade costs. Gao (2007) pointed out that, recently, manufacturing industries shifted from developed to developing countries. Before the move of manufacturing industries, manufacturing firms agglomerated in developed countries, which supported the high growth rates in those countries.

With these movements, the growth rates have followed non-monotonic trends. From 1950s to 1980s, Japan experienced rapid growth. However, after the 1980s, the growth rate of Japan declined, and other Asian countries, such as China, experienced rapid growth processes. However, the average growth rates of Asian countries in the 1990s fell, and, later, the growth rates increased. The Japan Cabinet Office SNA reported that, in 1956-1973, the average Japanese growth rate was 9.1% annually and, in 1974-1990, 4.2% annually. In these years, Japan

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1 OECD (2010) also reported that the R&D expenditure of Russia had increased, becoming 2.8% of OECD’s total R&D expenditure, in a scale similar to that in Canada or Italy.
grew rapidly. In 1991-1995, average growth rates dropped to 1.4%, and, in 1996-2000, they were 1.0%. On the other hand, China started its rapid economic growth in the 1980s. The World Bank (2011) reported that, in 1981-1990, the average growth rate of China was 9.2% annually and, in 1991-95, 12.9% annually. However, in 1996-2000, the average growth rate of China decreased and was 8.6%. Therefore, in 1996-2000, the economic growth in both Japan and China declined. However, in 2001-2005, China’s average growth rate increased to 9.8%, and, in 2006-2010, the average growth rate of China was 11.2%. Therefore, the growth rates of China and Japan followed non-monotonic movements. In 1950-1980, agglomeration of manufacturing firms in Japan promoted the high growth rates. In 1990s, hollowing out of manufacturing firms from Japan to other Asian countries started and progressed. In 2000s, agglomeration of manufacturing industries in Asian countries, such as China, supported high growth rates. Our model explains the mechanism of such non-monotonic movements of the growth process.

The focus of this paper is to study the influences of a reduction in trade costs on the location of manufacturing and innovation activities. In addition, in this paper, we explain the decline in trade costs and its large impact on economic growth. The main objective in this paper is to explain the mechanism behind the hollowing out of the manufacturing industry and innovation activities from developed to developing countries with a decline in trade costs after manufacturing agglomeration and rapid growth in the developed country. In our paper, we point out that market size and wage differential play important roles to express the mechanism behind the location shift of the manufacturing sector and the innovation sector with the reduction of trade costs.

To study the above facts, a Grossman-Helpman (1991) and Romer (1990)-type endogenous growth model with two countries is developed. In our model, there are three sectors, agriculture, manufacturing, and innovation. We assume that labor productivity in the agricultural sector differs in the two countries, which leads to different equilibrium wages in the two countries. In the agricultural sector, homogenous goods are produced only by labor with constant return production functions. These homogenous goods are traded internationally with no trade costs. Therefore, the price of these goods is the same in both countries. We assume that labor productivity in the agricultural sectors differs in the two countries. As a result, the equilibrium wages in the two countries differ.

In the standard international trade theory, there are comparative advantages between the two countries. On the other hand, in the standard new economic geography literature, such as that by Fujita, Krugman, and Venables (1999), market size plays an important role: manufacturing firms agglomerate in the country with a large market. In our paper, we examine the effects of comparative advantage and market size on the manufacturing location and innovation sector.

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2 The average growth rate of Japan was 1.3 in 2001-2005.
3 Puga and Venables (1996) and Fujita and Mori (1999) showed that Japan’s high wages induce the spread of manufacturing firms from Japan to East Asia.
4 Baldwin et al. (2003) studied both the effects of cost asymmetries and market size on
If wages in the large-market country are higher than those in the other country, the higher wages will lower the share of manufacturing firms in the large-market country. We assume that trade costs are initially high and gradually decrease. In this case, the share of manufacturing firms in the large-market country increases with a decline in trade costs. In a range of trade costs, we observe full agglomeration in the large-market country. Then, when trade costs are high, the effect of market size is larger than the effect of wage differential. In 1950-1970, since trade costs were relatively high, the large market size of Japan induced agglomeration of the manufacturing industry, which progressed with the reduction of trade costs.

However, when trade costs decrease, full agglomeration in the large-market country is broken, and some manufacturing firms switch their location from the large-market and high-wage to the small-market and low-wage country. Finally, we observe that all manufacturing firms agglomerate in the low-wage country when trade costs are very low. When trade costs are low, the effect of wage differential becomes larger than the effect of market size. Therefore, manufacturing firms switch location from a high-wage country to a low-wage country with the reduction of trade costs. In 1970-2010, the high wages in Japan induced the move of manufacturing firms from Japan to East Asia. In our paper, we point out that recent shifts of manufacturing firms from developed to developing countries have been induced by wage difference.

In addition, our model can explain the location shift of the innovation sector. In our paper, following Martin and Ottaviano (1999), there is local knowledge spillover from manufacturing firms to the innovation sector in a country. We assume that the innovation costs in a particular country decrease as the number of firms locating there increases. The innovation sector locates in the country that provides lower innovation costs. Then, the innovation sector tends to locate in the country with a large number of manufacturing firms. In the above process, the number of manufacturing firms is large in the large-market and high-wage country when trade costs are high. On the other hand, the number of manufacturing firms in the small-market and low-wage country increases when trade costs are low. Therefore, with the reduction of trade costs, the innovation sector changes its location. When trade costs are high, the innovation sector locates in the large-market country. When trade costs become low, the innovation sector shifts location to the small-market and low-wage country. These results mean that, in recent years, the move of manufacturing firms from Japan to East Asia has induced the location shift of the innovation sector.

The growth rate moves non-monotonously in this process. The growth rate is increased by the number of firms of the same country because of the spillover of local knowledge. Our model demonstrates that not only the market size but also the production costs of the manufacturing sector determine the location of the innovation sector, which derives economic growth. When trade costs are high, the reduction of trade costs results in the agglomeration of manufacturing firms in the large-market country. The growth rate then increases with the decline manufacturing location in the context of new economic geography.
in trade costs. When trade costs become sufficiently low, manufacturing firms start to shift their location from the large-market and high-wage country to the small-market and low-wage country. In this process, the reduction of trade costs lowers the growth rates. At an value of trade costs, innovation sector shift its location from the large-market and high-wage country to the small-market and low-wage country. After this point, the reduction of trade costs raises the growth rate of the economy.\footnote{Although it has been reported in many studies that trade liberalization results in increased growth rates, there are cases in the process of economic development in which the regulation of trade has resulted in increased growth rates. For example, Komiya et al. (1984) argued that the Japanese government regulated trade for the protection and expansion of domestic industry between 1950 and 1970. Due to this regulation, Japan achieved rapid growth. Therefore, there are cases in which trade liberalization lowers the growth rate. By introducing a wage differential, we can explain why both situations occur in the real world.}

Here, we mention welfare and policy implication of our model. In our model, decline in trade costs improves welfare of the whole economy, if growth rates are raised with decline in trade costs. However, it is possible that welfare of a country decreases with globalization (decline in trade costs), even if globalization fosters economic growth. With the decline in trade costs, location of firms changes: firms shift location from a country to the other country. Then welfare of a country which loses firms may decreases with globalization, while globalization improves the welfare of a country which get firms. In this case, there are conflicts between two countries: one country assumes to apply policies which lowers trade costs, such as Free Trade Agreement (FTA), and while the other country oppose to the policies. In the case that decline in trade costs raises the growth rates, coordination of the policies to lower trade costs improves the welfare of the whole economy. In that case, organization like WTO is necessary for the economy to achieve at the high growth rates. Our model, pointed out the above welfare and policy implication.

Some studies have examined the endogenous growth-new economic geography models, such as those by Baldwin, Martin, and Ottaviano (2001), Martin and Ottaviano (1999), (2001), Yamamoto (2003), Hirose and Yamamoto (2007), and Minniti and Parello (2011). In our paper, we expand on the ideas of Martin and Ottaviano (1999); in our model, wages in the two countries differ. In this paper, we show that both wage differences between two countries and market size differences have important effects on the location share of manufacturing firms and the innovation sector. In our paper, we show the mechanism of the process through which the location of the manufacturing firms, innovation sector, and economic growth shifts. Thus, our paper has rich implications and presents the mechanism of the history of economic development and agglomeration.

This paper is organized as follows. The next section is a presentation of the basic model. Section 3 is an analysis of the model and a presentation of the steady state equilibrium. This section presents a study of the effects of a decline in trade costs on the location of manufacturing firms, the innovation sector, and economic growth. Section 4 is the conclusion.
2 The Model

In this section, we introduce the model. There are two countries, 1 and 2. Variables referring to 1 have the subscript 1, and those referring to 2, the subscript 2. Each country is endowed with a fixed amount of labor, $L_1$ and $L_2$ ($L_1 > L_2$). Thus, country 1 has a larger amount of labor than country 2. This also means that country 1 has a larger market than country 2. Labor can be used to produce homogenous agricultural goods, differentiated manufactured goods, and blueprints. While labor can be mobile between sectors in the same country, it cannot be mobile between different countries. For variety to be achievable, a blueprint has to be developed. The blueprint is then protected by a patent that cannot expire. Once the blueprints are available, the patent can be sold to any firm located in either country. The innovation and the production process are, therefore, conducted by different economic agents and, possibly, in different countries.

The intertemporal utility function of the consumer in country $s$ ($s = 1, 2$) is as follows:

$$U_s = \int_0^\infty e^{-\rho t} (Y_{st} + \mu \log M_{st}) \, dt,$$

where

$$M_{st} = \left[ \int_0^{n_{1t}} m_{1st} (i)^{\frac{\sigma - 1}{\sigma}} \, di + \int_0^{n_{2t}} m_{2st} (i)^{\frac{\sigma - 1}{\sigma}} \, di \right]^\frac{1}{\sigma - 1}, \, \sigma > 1.$$  

Here, $Y_{st}$ is the consumption of agriculture goods at time $t$, $M_{st}$ is the consumption of the composite of manufactured goods at time $t$, $\rho$ is the subjective discount rate, and $\mu$ is the constant parameter. $m_{rst} (i)$ denotes the consumption of the variety $i$ th manufactured goods produced by a firm in country $r$ ($r = 1, 2$). $n_{rt}$ is the number of varieties produced by firms in country $r$ at time $t$. $N_t = n_{1t} + n_{2t}$ denotes the total number of varieties at time $t$. $\sigma$ is the constant parameter that represents the elasticity of substitution among differentiated goods. Following Grossman and Helpman (1991), the market has been characterized by free financial movements between two countries. Thus, the interest rate of both countries is the same at all times ($r_{1t} = r_{2t} = r_t$). The intertemporal optimization behavior of the consumer brings about the next equation

$$r_t = \rho.$$  

We can derive the following instantaneous demand functions (we take homogeneous goods as the numeraire, such that $p_A = 1$),

$$M_{st} = \frac{\mu}{P_{st}},$$  

$$P_{st} = \left( \int_0^{n_{1t}} p_{1st}(i)^{1-\sigma} \, di + \int_0^{n_{2t}} p_{2st}(i)^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}},$$  

$$Y_{st} = E_{st} - \mu.$$
\[ m_{rst} (i) = \frac{\mu P_{st}^{\sigma-1}}{p_{rst} (i)^{\sigma}}, \]

where \( P_{st} \) is called the ‘price index’ in country \( s \) at time \( t \). \( p_{rst} (i) \) is the consumer price of variety \( i \), which is produced in \( r \) and consumed in \( s \), and \( E_{st} \) represents the instantaneous expenditure of a consumer in country \( s \) at time \( t \).

Here, we describe the condition of the homogenous goods sector. A homogeneous agriculture goods market is perfectly competitive. We assume that the international trade of homogenous goods incurs no trade costs. Thus, the price of the homogenous goods is equalized across countries.

We assume that the productivities of labor in the agricultural sector differ between the two countries. In country 1, \( a_1 \) units of agriculture goods are produced with 1 unit of labor. In country 2, 1 unit of labor produces \( a_2 \) units of agricultural goods. We assume that the international trade of homogenous goods incurs no trade costs. Therefore, since we assume that agricultural goods are produced in both countries at the equilibrium\(^6\), the equilibrium wages in the two countries become \( w_1 = a_1 \), \( w_2 = a_2 \).

In the manufacturing goods sector, manufacturing firms operate under Dixit-Stiglitz (1977)-type monopolistic competition. Each firm produces differentiated goods, and each variety is produced by one firm. To start a production activity, a firm in country \( r \) is required to buy one unit of a patent produced by the innovation sector at market price \( v_{rt} \), which plays the role of fixed costs for the firms. Moreover, a firm locating in a country uses \( c \) units of labor in its country as the marginal input to produce one unit of manufactured goods. Potential firms can freely enter a production activity as long as the operating profits are positive and can choose to locate in a country where profits are higher. Under this production structure, each manufacturing firm sets the following constant markup price:

\[ p_r = \frac{\sigma}{\sigma - 1} \cdot c w_r = \frac{\sigma}{\sigma - 1} \cdot c a_r, \ r = 1, 2. \]  

\( c w_r = c a_r \) represents the marginal costs for manufacturing firms. The international trade of manufactured goods incurs ‘iceberg’-type trade costs. If a firm sends \( x \) units of goods to a foreign country, it must dispatch \( \tau x \) units of goods. \( \tau > 1 \) represents the trade costs. Thus, consumer prices are \( p_{rs} = p_r \) if \( r = s \), and \( p_{rs} = \tau p_r \) if \( r \neq s \). With constant markup pricing, the operating profit of each firm and the price index are written as,

\[ \pi_{rt} = \frac{c w_r}{\sigma - 1} q_{rt} = \frac{c a_r}{\sigma - 1} q_{rt}, \]

\[ P_{it} = \frac{\sigma c}{\sigma - 1} \cdot \left( n_{it} a_i^{1-\sigma} + n_{jt} \phi a_j^{1-\sigma} \right)^{\frac{\sigma}{\sigma - 1}}, \ i = 1, 2, \ j = 1, 2, \ i \neq j, \]

where \( \phi \equiv \tau^{1-\sigma} \in (0, 1) \) represents the freeness of trade.

\(^6\)We assume that \( \mu \) is sufficiently small so that the total demand for agricultural goods is sufficiently large, that is, \((E_{1t} - \mu) L_1 + (E_{2t} - \mu) L_2 \geq a_1 L_1 \) and \((E_{1t} - \mu) L_1 + (E_{2t} - \mu) L_2 \geq a_2 L_2 \). In this model, \( E_{1t} \geq a_1 \) and \( E_{2t} \geq a_2 \). Therefore, we assume that \((a_1 - \mu) L_1 + (a_2 - \mu) L_2 \geq a_1 L_1 \) and \((a_1 - \mu) L_1 + (a_2 - \mu) L_2 \geq a_2 L_2 \).
We assume that the capital market is perfectly competitive. We assume that there are risk-free assets and their interest rate is \( r_t \). The value of the firm (which is the market price of the patent) is equalized to the present value of the sum of discounted profit over time. From (9), it represents

\[
v_{rt} = \int_t^\infty e^{-r(i-t)} \cdot \frac{c\sigma}{\sigma - 1} q_{rt} dt.
\]

(11)

Differentiating (11) with respect to \( t \), we obtain the no-arbitrage condition for capital investment \( v_{rt} \):

\[
\frac{c\sigma}{\sigma - 1} \cdot q_{rt} + \dot{v}_{rt} = rv_{rt}.
\]

(12)

In the innovation sector, we assume that innovation firms produce 1 unit of patent by using \( I_s \) units of labor. For innovators in country \( s \), the innovation costs for a patent are written as \( w_s I_{st} (s = 1, 2) \). Innovators choose their own location with no relocation cost. Then, innovators choose \( s \), where they can minimize innovation costs \( w_s I_{st} \). If \( w_1 I_{1t} < (>) w_2 I_{2t} \), then the innovator locates in country 1(2). We assume that \( I_s \) depends on the number of home and foreign varieties of manufacturing firms, as follows:

\[
I_{it} = \frac{\eta}{n_{it} + \delta n_{jt}}, \quad i = 1, 2, \quad j = 1, 2, \quad i \neq j,
\]

(13)

where \( \delta \in (0, 1) \) represents the degree of international knowledge spillover. In this innovation technology, the agglomeration of manufacturing firms in a country lowers the innovation costs in this country, since we assume that \( \delta \in (0, 1) \). Let us define \( s \equiv \frac{n_1}{n_1 + n_2} \) as the share of manufacturing firms in country 1. We can check \( \frac{dw_1 I_{1t}}{ds} < 0 \) and \( \frac{dw_2 I_{2t}}{ds} > 0 \) as \( N \equiv n_1 + n_2 \) fixed. A cost-minimizing innovator chooses its location according to the share of manufacturing firms in country 1, \( s \equiv n_1/N \), and the relative wage in country 1 to country 2, \( \frac{a_1}{a_2} \). Therefore, we can describe the location behavior of innovators as follows:

The location of the innovator is

\[
\begin{cases} 
\text{country 2} & \text{if } 0 \leq s \leq \hat{s}, \\
\text{country 1} & \text{if } \hat{s} \leq s \leq 1
\end{cases}
\]

(14)

where \( \hat{s} \equiv \frac{n_1 - a_2 \sigma}{(1-\delta)(a_1 + a_2)} = \frac{n_1}{a_2 + \delta (a_1 + a_2)} \cdot \frac{1+\delta}{\frac{a_2}{a_1} + \frac{1}{\frac{a_2}{a_1} + 1}} \). As the productivity for agricultural goods of country 1 relative to that of country 2 becomes larger, wages in country 1 become higher relative to those in country 2. Thus, if \( a_1/a_2 \) is sufficiently large, innovation firms locate in country 2. On the other hand, if the share of manufacturing firms in country 1 is large, positive technological externality lowers the innovation costs in country 1. Then,

\( ^7 \)In Hirose and Yamamoto (2007), knowledge spillover from a foreign country is asymmetric in two countries. We can extend our model with the assumption of asymmetric knowledge spillover.

\( ^8 \)In our model, innovation costs decrease with numbers of firms which are flow variables. This assumption follows Martin and Ottaviano (1999). If we follow Grossman and Helpman (1991) and innovation costs decrease with past innovation experiences, location of innovation are not influenced by manufacturing location.
if country 1 absorbs many manufacturing firms relative to country 2 (s is large), the innovation sector locates in country 1. Figure 1 depicts the relationship among relative wage, manufacturing location, and location of innovation sector.

3 Equilibrium conditions and steady states

3.1 Equilibrium location of manufacturing firms

In this section, we analyze the model and present the equilibrium conditions. The total production quantities of a firm of each variety in countries 1 and 2 must satisfy $q_1 = L_1m_{11} + \tau L_2m_{12}$ and $q_2 = L_2m_{22} + \tau L_1m_{21}$, respectively. Introducing (7), (8), and (5), the market-clearing conditions for each variety of goods are

$$q_{11}(s) = a_1\sigma \cdot \frac{L_1}{\sigma cN_t} \cdot \mu \left( \frac{L_1}{s_1a_1^{-\sigma} + (1 - s_1)\phi a_1^{-1+\sigma}} + \frac{\phi L_2}{(1 - s_1)a_1^{-1+\sigma} + s_1\phi a_1^{-1+\sigma}} \right)$$

$$q_{21}(s) = a_2\sigma \cdot \frac{L_2}{\sigma cN_t} \cdot \mu \left( \frac{L_2}{(1 - s_1)a_2^{-1+\sigma} + s_1\phi a_2^{-1+\sigma}} + \frac{\phi L_1}{s_1a_1^{-1+\sigma} + (1 - s_1)\phi a_2^{-1+\sigma}} \right).$$

For the following analysis, we define $A \equiv \left( \frac{a_1}{a_2} \right)^{1-\sigma}$ and $\lambda = \frac{L_1}{L_1 + L_2}$. With this definition, we can see that $\frac{dA}{d\left(\frac{a_1}{a_2}\right)} < 0$, $\lim A = \infty$, $\lim A = 0$ and $1/2 < \lambda < 1$.

We must clarify the parameter conditions in which the firm's location equilibrium becomes an interior solution or a corner solution. The difference of operating profits across two countries is

$$\Phi(s) \equiv \pi_1 - \pi_2 = \frac{\mu}{\sigma N_t} \left[ \frac{(A - \phi)\lambda}{sA + (1 - s)\phi} + \frac{(\phi A - 1)(1 - \lambda)}{(1 - s) + s\phi A} \right].$$

A manufacturing firm determines its location taking s as given. When $\Phi(s) > (<) 0$, a firm locates in country 1 (2). When $\Phi(s) = 0$, it is indifferent for a firm whether it locates in country 1 or 2. Since entrance and exit of manufacturing firms are free, firms enter into one country as long as its profits are positive and higher than those of the other country. We define $s^*$ as an equilibrium value of s. First, if $\Phi(0) \leq 0$, then $s^* = 0$. Second, if $\Phi(1) \geq 0$, then $s^* = 1$. Third, if $\Phi(0) > 0$ and $\Phi(1) < 0$, then $s^*$ is an interior solution ($0 < s^* < 1$) and $\Phi(s^*) = 0$. To determine which equilibrium is realized, we analyze the model by identifying three cases: (a) $0 < A \leq \phi$, (b) $\frac{1}{\phi} \leq A < \infty$, and (c) $\phi < A < \frac{1}{\phi}$.

(a) The case of $0 < A \leq \phi$ leads to both $A - \phi \leq 0$ and $\phi A - 1 < 0$ in the numerators of (17). Thus, $\Phi(s) < 0$ for all $s \in [0, 1]$ and for all $\lambda \in \left(\frac{1}{2}, 1\right)$. Then, $\Phi(0) < 0$ for all $\lambda \in \left(\frac{1}{2}, 1\right)$. This means that $s^* = 0$. Then, when $0 < A \leq \phi$, all manufacturing firms agglomerate in country 2.
Otherwise, it has corner solutions. In other words, if interior solutions in some parameter conditions: the sign of (17) is not clear. However, since (17) is a decreasing function of \( \frac{1}{\phi} \), and, if \( \phi A - 1 \geq 0 \) in the numerator of (17). Thus, \( \Phi(s) > 0 \) for all \( s \in [0, 1] \) and for all \( \lambda \in \left( \frac{1}{2}, 1 \right) \). Then, \( \Phi(1) > 0 \) for all \( \lambda \in \left( \frac{1}{2}, 1 \right) \). This means \( s^* = 1 \). Therefore, in the case of \( \frac{1}{\phi} \leq A < \infty \), all manufacturing firms agglomerate in country 1.

(c) The case of \( \phi < A < \frac{1}{\phi} \) leads to both \( A - \phi > 0 \) and \( \phi A - 1 < 0 \). The sign of (17) is not clear. However, since (17) is a decreasing function of \( \lambda \), it has interior solutions in some parameter conditions: \( \Phi(0) > 0 \) and \( \Phi(1) < 0 \). The conditions of \( \Phi(0) > 0 \) and \( \Phi(1) < 0 \) are equivalent to (0) \( \frac{\phi(1-\phi A)}{A(1+\phi)(1-\phi)} < \lambda < \frac{1+\phi A}{(1+\phi)(1-\phi)} \) (1\(< \)). Under this condition, \( s^* \) satisfies \( 0 < s^* < 1 \) and \( \Phi(s^*) = 0 \). Otherwise, it has corner solutions. In other words, if \( \lambda \leq \frac{\phi(1-\phi A)}{A(1+\phi)(1-\phi)} \), then, \( s^* = 0 \), and, if \( \lambda \geq \frac{1-\phi A}{(1+\phi)(1-\phi)} \), then, \( s^* = 1 \).

From the above discussion ((a), (b), and (c)), the equilibrium share of firms in country 1 is

\[
s^*(\lambda, A, \phi) = \begin{cases} 
0 & \text{if } (\lambda, A, \phi) \in \{ \lambda, A, \phi|0 \leq \lambda \leq \lambda_0, 0 < A < 1 \} \\
\frac{A(1+\phi)(1-\phi)\lambda - \phi(1-\phi A)}{(1-\phi A)(A-\phi)} & \text{if } (\lambda, A, \phi) \in \{ \lambda, A, \phi|\lambda_0 < \lambda \leq 1, \phi < A < \frac{1}{\phi} \} \\
1 & \text{if } (\lambda, A, \phi) \in \{ \lambda, A, \phi|\lambda_1 < \lambda \leq 1, 0 < A < 1 \} 
\end{cases}
\]

(18)

where \( \lambda_0 \equiv \frac{\phi(1-\lambda A)}{A(1+\phi)(1-\phi)} \), \( \lambda_1 \equiv \frac{1-A\phi}{(1+\phi)(1-\phi)} \).

By differentiating an interior solution with respect to \( \lambda \) and \( A \), we obtained the following lemma.

**Lemma 1** (1) The interior solution of \( s^*(\lambda, A, \phi) \) is an increasing function of \( \lambda \). That is, \( \frac{\partial s^*}{\partial \lambda} > 0 \). (2) The interior solution of \( s^*(\lambda, A, \phi) \) is an increasing function of \( A \). That is, \( \frac{\partial s^*}{\partial A} > 0 \).

**Proof:** (1) \( \frac{\partial s^*}{\partial \lambda} = \frac{A(1+\phi)(1-\phi)\lambda - \phi(1-\phi A)}{(1-\phi A)(A-\phi)} \geq 0 \) (Note 1 - \( \phi A > 0 \) and \( A - \phi > 0 \)). (2) See Appendix 1.

Lemma 1 says that the share of firms in country 1 rises if (1) the relative market size of country 1 becomes larger or if (2) the relative wage in country 1 becomes lower.

Here, we study the effect of decline in trade costs on the location of manufacturing firms. To study the effects of trade costs on the location of manufacturing firms, with \( \lambda \) and \( A \) fixed, we check the sign of \( \frac{\partial s^*}{\partial A} \). We obtain the following proposition by calculations shown in the Appendix 2. Here, we define \( \phi_1 \equiv \frac{A-(A^2-4\lambda(1-\lambda))^{1/2}}{2\lambda}, \phi_2 \equiv \frac{A+(A^2-4\lambda(1-\lambda))^{1/2}}{2\lambda}, \) and \( \phi_3 \equiv \frac{1-(1-4A^2\lambda(1-\lambda))^{1/2}}{2A(1-\lambda)} \).

We can observe that \( \phi_1 < \phi_2 < \phi_3 \). In addition, we assume that \( A < 1 \). This assumption means that the large-market country is the high-cost country for manufacturing firms.

**Proposition 1** (1) Let us consider that the parameters satisfy \( \lambda \geq \frac{1}{1+\phi^2} \). \( s^* \) is an increasing function of \( \phi \) when \( 0 \leq \phi \leq \phi_1 \). On the other hand, \( s^* \) is
a decreasing function of \( \phi \) when \( \phi_2 \leq \phi \leq \phi_3 \). (2) In the case in which the parameters satisfy the relation \( \lambda \geq \frac{1+(1-A^2)^{1/2}}{2} \), when \( \phi_1 \leq \phi \leq \phi_2 \), full agglomeration in country 1 is realized. On the other hand, when \( \phi_3 \leq \phi \leq 1 \), all firms agglomerate in country 2.

**Proof.** See Appendix 2. ■

The above proposition says that, when trade costs are high, the market-size effect dominates the cost advantage: the large-market country attracts manufacturing firms with decline in trade costs. On the other hand, when trade costs become low, the cost advantage dominates the market-size effect.

**Proposition 2** When trade costs are high \((0 \leq \phi \leq \phi_2)\), the large-market country (country 1) attracts firms with the decline in trade costs. On the other hand, when trade costs become low \((\phi_2 \leq \phi \leq 1)\), in the low-cost country (country 2), agglomeration of manufacturing firms progresses with the decline in trade costs.

Propositions 1 and 2 say that, when trade costs are high, declining trade costs facilitate agglomeration to the large-market (and higher-wage) country. On the other hand, when trade costs are small, declining trade costs foster agglomeration in the small-market (and lower-wage) country. The reduction of trade costs has two effects. One is that locating in the larger-market country becomes more profitable because firms can transport manufactured goods with lower trade costs. Firms, then, need not locate in the small-market country. This effect is reported in many studies of new economic geography, such as those by Martin and Ottaviano (1999, 2001). Another effect is that, when trade costs become lower, the market size becomes less important, and the wage difference becomes more important for a firm’s location choice.\(^9\) Indeed, in the \( \phi = 1 \) case, i.e., perfect free trade, all manufacturing firms agglomerate in country 2, where wages are lower, i.e., \( s^* = 0 \) for every \( \lambda \in (\frac{1}{2}, 1) \) and \( A \in (0, 1) \). When trade costs are high, the former effect dominates over the latter effect, and then \( s^* \) increases. With a decline in trade costs, the difference in the two effects becomes small; then, the latter effect dominates over the former effect. As a result, \( s^* \) decreases. Finally, at very low trade costs, all manufacturing firms agglomerate in country 2 (Figure 2).

These results shows that in 1950-1970, since trade costs were relatively high, the large market size of developed country (Japan) induced agglomeration of the manufacturing industry, which progressed with the reduction of trade costs. However, when trade costs decrease, full agglomeration in the large-market country is broken. Some manufacturing firms switch their location from the high- to the low-wage country. Finally, we observe that all manufacturing firms agglomerate in the low-wage country when trade costs are very low. When trade costs are low, the effect of wage differential becomes larger than the effect of market size. Therefore, manufacturing firms switch location from a high-wage

\(^9\) In Gao (2007), because globalization makes the manufacturing/agriculture wage ratio in country 2 higher and manufacturing labor supply becomes larger, the manufacturing firms in country 2 expand, with a decline in trade costs.
country to a low-wage country with the reduction of trade costs. In 1970-2010, the high wages in Japan induced the move of manufacturing firms from Japan to East Asia. Results in this subsection shows that recent shifts of manufacturing firms from developed to developing countries have been induced by wage difference.

3.2 Equilibrium location of the innovation sector

The innovation firm determines its location according to $s$ and $a_1/a_2$ as (14). Some calculations lead us to the next lemma.

Lemma 2 (1) When $A > 1$, $s^* > \frac{1}{2}$, and $\hat{s} < \frac{1}{2}$ are always satisfied. (2) When $(\frac{1}{2})^{1-\sigma} < A < 1$, $\frac{1}{2} < \hat{s} < 1$, and $\partial \hat{s} / \partial A < 0$. (3) When $A < (\frac{1}{2})^{1-\sigma}$, $1 < \hat{s}$.

Proof. (1) (18) leads to $s^* - \frac{1}{2} > 0$. (14) leads to $\hat{s} - \frac{1}{2} < 0$. Thus, we see that $\hat{s} < s^*$. This means that the innovation sector locates in country 1.

(2) From $\hat{s} = -\frac{s}{A} \frac{(1-\sigma)(\frac{a_1}{a_2} - \sigma)}{2(1-\sigma)} > 1$ when $(\frac{1}{2})^{1-\sigma} \leq A < 1$. Moreover, $\frac{\partial \hat{s}}{\partial (\frac{s}{A})} = \frac{\frac{1}{2} - \sigma}{(1-\sigma)(\frac{a_1}{a_2} - \sigma)} > 0$ means that $\partial \hat{s} / \partial A < 0$. (3) If $A \leq (\frac{1}{2})^{1-\sigma}$, then $\hat{s} = -\frac{s}{A} \frac{(1-\sigma)(\frac{a_1}{a_2} - \sigma)}{2(1-\sigma)} > 1$. ■

Next, we study the effects of trade costs on the location of the innovation sector. From (14), with $a_1/a_2 (\geq 1)$ fixed, the innovator determines its location according to $s^*$. Then, we can draw $\hat{s}$ line horizontally between $\frac{1}{2}$ and 1 as in Figure 2. By taking $\lambda$ and $A$ such that $\lambda \geq \frac{1}{1+\lambda A}$, $\partial s^* / \partial \phi |_{\phi=\hat{s}}$ satisfied, which we show in Appendix 2. Here, when $\phi \approx 0$, $s^* \approx \lambda$. Therefore, $\lambda \geq \frac{1}{1+\lambda A}$ and $\lambda \geq \hat{s}$ mean that when trade costs are high, the innovation sector locate in country 1. We assume that $\delta > \frac{1-A \frac{1-\phi_1^2}{1+\phi_1 A}}{A^{1+\sigma} - A^2}$. In Appendix 3, we show that when $\delta > \frac{1-A \frac{1-\phi_1^2}{1+\phi_1 A}}{A^{1+\sigma} - A^2}$, $\hat{s} > \frac{1}{1+\lambda A^2}$. Thus under our assumption of $\delta > \frac{1-A \frac{1-\phi_1^2}{1+\phi_1 A}}{A^{1+\sigma} - A^2}$, $\lambda \geq \hat{s}$ involves $\lambda \geq \frac{1}{1+\lambda A^2}$. 10

If $\lambda \geq \frac{1+(1-A^2)^{\frac{1}{2}}}{2}$, with a decline in trade costs, full agglomeration in country 1 ($s^* = 1$) is realized when $\phi_1 \leq \phi \leq \phi_2$. Moreover, with a decline in trade costs, full agglomeration in country 1 is broken, and manufacturing firms change their location from country 1 to country 2. At $\phi$, the share of manufacturing firms falls below the threshold, and the innovation sector moves from country 1 to country 2. We summarize the discussion above as the next proposition:

10If we assume that $\delta < \frac{1-A \frac{1-\phi_1^2}{1+\phi_1 A}}{A^{1+\sigma} - A^2}$, results of the paper are not changed. In the case of $\delta < \frac{1-A \frac{1-\phi_1^2}{1+\phi_1 A}}{A^{1+\sigma} - A^2}$, $\lambda \geq \frac{1}{1+\lambda A^2}$ involves $\lambda \geq \hat{s}$. In this case, we substitute $\lambda \geq \frac{1}{1+\lambda A^2}$ into $\lambda \geq \hat{s}$ in propositions which will appear in later propositions.
Proposition 3 Let us consider that the parameters satisfy $\lambda \geq \hat{s}$ and $(\frac{1}{\sigma})^{1-\sigma} \leq A < 1$. When trade costs are high ($0 \leq \phi < \hat{\phi}$), the innovation sector locates in the large-market country (country 1). On the other hand, when trade costs are low, ($\hat{\phi} \leq \phi \leq 1$), the innovation sector locates in the low-cost country (country 2).

In Figure 2, we depict the equilibrium location of innovation sector. Results in this subsection show that innovation sector follows the move of manufacturing firms. When trade costs are high, innovation sector locates in the large-market country, since the large market attracts many manufacturing firms. However, when trade costs become low, innovation sector locates in the low-wage country, since the role of wage differential dominates the market-size effect. In recent years, the move of manufacturing firms from Japan to East Asia has induced the location shift of the innovation sector.

3.3 Growth rates

As in the study by Grossman and Helpman (1991), this model has a unique steady state in which the mass of variety is expanding at a constant rate over time. Since the innovation sector is assumed to be under free entry, at the equilibrium, the innovation cost must be equalized to the value of the patent. Then, $v_t = \frac{\eta a_1 N_t}{N_t (s^* + \delta (1-s^*) \hat{\phi})}$ when the innovation sector locates in country 1. $s^*$ is constant over time from (18), and $v$ must decrease at the same rate as $N_t$ increases; thus, $\frac{\delta v}{v_t} = -\frac{N_t}{N_t}$. At the equilibrium, the total sales of a manufacturing firm are $a_1 q_1 = a_1 q_2 = \frac{\mu (\sigma-1) (L_1+L_2)}{\sigma a N_t}$. Introducing these equalities into (12), we derive the growth rate at steady states (we define $g$ as the rate of variety expanding at steady states, that is, $g \equiv \frac{N_t}{N_t}$).

$$g(s, \gamma \left( \frac{a_1}{a_2} \right), a_1 + a_2) = \left\{ \begin{array}{ll} \frac{\mu (L_1 + L_2)}{\sigma} \frac{(1-s) + \delta s}{\eta (1-s) (a_1 + a_2)} - \rho & \text{if the innovator locates in country 2.} \\ \frac{\mu (L_1 + L_2)}{\sigma} \frac{1}{\eta (a_1 + a_2)} - \rho & \text{if the innovator locates in country 1.} \end{array} \right.$$ (19)

where $\gamma \equiv \frac{a_1}{a_2} = \frac{a_1}{a_1 + a_2}$.

First, differentiating (19) with respect to $s$

$$\frac{\partial g}{\partial s} = \left\{ \begin{array}{ll} \frac{\mu (L_1 + L_2)}{\sigma} \frac{-s + \delta (1-s)}{\eta (1-s) (a_1 + a_2)} < 0 & \text{if the innovator locates in country 2.} \\ \frac{\mu (L_1 + L_2)}{\sigma} \frac{(1-s) - s}{\eta (a_1 + a_2)} > 0 & \text{if the innovator locates in country 1.} \end{array} \right.$$ (20)

The sign of $\frac{\partial g}{\partial s}$ depends on the location of the innovation sector. The next lemma represents this property.

Lemma 3 The agglomeration of manufacturing firms in the country where the innovation sector locates enhances the economic growth rate.
Let us assume that the innovation sector locates in country 2. When the agglomeration of a manufacturing firm in country 1 progresses, the unit requirement for producing a patent in country 2 increases, since international knowledge spillover is imperfect. The growth rates then become lower. Conversely, when the innovation sector locates in country 1, agglomeration in country 1 makes the innovation sector more efficient. The growth rates then become higher.

We study the economic growth rate at a steady state in the process of the decline in trade costs. Lemma 3 means that agglomeration in the country where the innovation sector locates fosters the growth rate. Thus, when the innovation sector locates in country 1, the growth rate moves in the same direction as \( s \) moves. When the innovation sector locates in country 2, the growth rate moves in the opposite direction against \( s \). In addition, at the full agglomeration case, the growth rate is as follows:

\[
g(1, \gamma \left( \frac{a_1}{a_2}, a_1 + a_2 \right)) = \frac{\mu (L_1 + L_2)}{\sigma} \cdot \frac{1}{\eta \gamma (a_1 + a_2)} - \rho,
\]

\[
g(0, \gamma \left( \frac{a_1}{a_2}, a_1 + a_2 \right)) = \frac{\mu (L_1 + L_2)}{\sigma} \cdot \frac{1}{\eta (1 - \gamma) (a_1 + a_2)} - \rho.
\]

From \( a_1 \geq a_2 \), we can see that \( \frac{1}{2} \leq \gamma < 1 \) and then \( 1 - \gamma \leq \gamma \). This implies \( g(1, \bullet) \leq g(0, \bullet) \). After summarizing this discussion, we obtain the next proposition:

**Proposition 4** Consider that the parameter condition of \( \lambda \geq \delta, \lambda \geq \frac{1+(1-A^2)^\frac{1}{2}}{2} \), and \( \left( \frac{1}{\delta} \right)^{1-\sigma} \leq A < 1 \) are satisfied. (1) When \( 0 \leq \phi \leq \phi_1 \) and \( \hat{\phi} \leq \phi \leq \phi_3 \), the decline in trade costs raises the growth rates. (2) When \( \phi_1 \leq \phi \leq \phi_2 \), the economic growth rate is \( g = g(1, \bullet) \). (3) When \( \phi_2 < \phi \leq \phi \), the economic growth rate decreases with the decline in trade costs. (4) When \( \phi_3 \leq \phi \leq 1 \), the economic growth rate becomes the maximum value: \( g = g(0, \bullet) \).

We can draw the relationship between trade costs and the growth rate as shown in Figure 3. When \( 0 \leq \phi \leq \phi_1 \), the decline in trade costs manifests the agglomeration of manufacturing firms in the large-market country, and the innovation sector locates in this country. Therefore, when \( 0 \leq \phi \leq \phi_1 \), the decline in trade costs raises the growth rate. When \( \phi_1 \leq \phi \leq \phi_2 \), all manufacturing firms agglomerate in the large-market country, and innovation sector exists in this country. Then, the growth rate is constant at the relatively high value. In the case of \( \phi_2 \leq \phi \leq \phi \), hollowing out of manufacturing firms from the large-market country to the low-cost country progresses with the decline in trade costs, while the innovation sector exists in this country. Thus, when \( \phi_2 \leq \phi \leq \phi \), the decline in trade costs lowers the growth rate. At \( \phi = \hat{\phi} \), the innovation sector switches their location from the large-market country to the low-cost country. When \( \hat{\phi} \leq \phi \leq \phi_3 \), the growth rate increases with agglomeration in the low-cost country that progresses with the decline in trade costs.
Finally, when $\phi_3 \leq \phi \leq 1$, all firms agglomerate in the country with low costs, and the growth rate becomes the maximum value.

Our model demonstrates that the market size and the production costs of the manufacturing sector determine the location of the innovation sector, which derives economic growth. When trade costs are high, the reduction of trade costs derives the agglomeration of manufacturing firms in the large-market country. The growth rate then increases with the decline in trade costs. When trade costs become sufficiently low, manufacturing firms start to shift their location from the large-market and high-wage country to the small-market and low-wage country. In this process, the reduction of trade costs lowers the growth rates. At an value of trade costs, innovation sector shift its location from the large-market and high-wage country to the small-market and low-wage country. After this point, the reduction of trade costs raises the growth rate of the economy.

Our model’s implication is as follows: In 1950-1980, the large market in Japan fosters agglomeration of manufacturing firms in Japan. Agglomeration of manufacturing firms in Japan promoted the high growth rates. In 1990s, hollowing out of manufacturing firms from Japan to other Asian countries started and progressed, since relative wages in Japan was high. In 2000s, agglomeration of manufacturing industries in Asian countries, such as China were progressed since wages in those countries were relatively low, and those manufacturing agglomeration supported high growth rates.

3.4 Effects of the change of the wage difference and the relative market size on the location of manufacturing firms and the growth rate

In this subsection, we study the effect of wage differences and the relative market size on the location of manufacturing firms and economic growth. The wage difference is represented by $\frac{a_1}{a_2}$, and the relative market size is expressed with $\lambda \equiv \frac{L_1}{L_1+L_2}$. Differentiating (19) with respect to $\frac{a_1}{a_2}$, we obtain:11

$$\frac{\partial g}{\partial \left( \frac{a_1}{a_2} \right)} = \partial g \left( \partial \gamma \right) \left( \frac{a_1}{a_2} \right) \right) = \begin{cases} \frac{\mu(L_1+L_2)}{\sigma} \cdot \frac{(1-s)+\theta s}{\eta(1-\gamma)^2(a_1+a_2)} \cdot \frac{\partial \gamma}{\partial \left( \frac{a_1}{a_2} \right)} > 0 \quad \text{if the innovator locates in country 2.} \\
\frac{\mu(L_1+L_2)}{\sigma} \cdot \frac{-(s+\theta(1-s))}{\eta \gamma^2(a_1+a_2)} \cdot \frac{\partial \gamma}{\partial \left( \frac{a_1}{a_2} \right)} < 0 \quad \text{if the innovator locates in country 1.} 
\end{cases}$$

Raising $\frac{a_1}{a_2}$ means that the relative wages in country 1 become higher. Thus, the innovation cost in country 1 becomes larger relative to that in country 2. Therefore, $\frac{\partial g \left( \frac{a_1}{a_2} \right)}{\partial \left( \frac{a_1}{a_2} \right)}$ is negative when the innovation sector locates in country 1. Conversely, $\frac{\partial g \left( \frac{a_1}{a_2} \right)}{\partial \left( \frac{a_1}{a_2} \right)}$ is positive when the innovation sector locates in country 2.

\[11\text{Notice that } \frac{\partial g \left( \frac{a_1}{a_2} \right)}{\partial \left( \frac{a_1}{a_2} \right)} > 0.\]
We substitute \( s = s^* \left( \lambda, A \left( \frac{a_1}{a_2} \right), \phi \right) \) characterized by (18) into (19),
\[
g^* \left( \lambda, A \left( \frac{a_1}{a_2}, a_1 + a_2, \phi \right) \equiv g \left[ s^* \left( \lambda, A \left( \frac{a_1}{a_2} \right), \phi \right), \frac{a_1}{a_2}, a_1 + a_2 \right]. \tag{21}
\]
We differentiate (21) with respect to parameters \( \lambda \) and \( \frac{a_1}{a_2} \).

From \( \frac{\partial g^*}{\partial x} = \frac{\partial g}{\partial x} \frac{\partial s^*}{\partial x} \) and \( \frac{\partial s^*}{\partial x} > 0 \), we obtain \( \frac{\partial g^*}{\partial x} < (\geq)0 \) when the innovation sector locates in country 2(1). This means that, if the market size of the country in which the innovation sector locates becomes larger, manufacturing firms become more agglomerated in that country. Lemma ?? means that the innovation activity becomes more efficient and growth rates become higher depending on the market size of the country where the innovation sector locates.

Differentiating (21) with respect to \( \frac{a_1}{a_2} \), we derive the following equations:
\[
\frac{\partial g^*}{\partial (a_1/a_2)} = \begin{cases} 
\frac{\partial g}{\partial s^*} \frac{\partial s^*}{\partial A} \frac{\partial A}{\partial (a_1/a_2)} + \frac{\partial g}{\partial (a_1/a_2)} > 0 \quad & \text{if the innovator locates in country 2.} \\
\frac{\partial g}{\partial s^*} \frac{\partial s^*}{\partial A} \frac{\partial A}{\partial (a_1/a_2)} + \frac{\partial g}{\partial (a_1/a_2)} < 0 \quad & \text{if the innovator locates in country 1.}
\end{cases}
\]

The change of relative wages has two effects on the growth rate. One is the direct effect, in which innovation costs change. The other is the indirect effect, in which the share of manufacturing firms changes. Because these two effects have the same direction, we can see the sign of \( \frac{\partial g^*}{\partial (a_1/a_2)} \). Let us assume that the innovator locates in country 1 and the relative wages in country 1 increase. The innovation costs become higher by the direct effect, and innovation technology becomes inefficient through the indirect effect of the share of firms in country 1. Therefore, the growth rates become lower when the innovation sector locates in country 1 and the relative wages in country 1 increase. Our studies in this subsection show that the wage difference and the relative market size play important roles in determining the growth rates. The rise in the wage difference between the high-cost and low-cost country lowers the growth rates when the innovation sector locates in the high-cost country; on the other hand, it raises the growth rates when the innovation sector locates in the low-cost country. The expansion of the relative market size raises the growth rate when the innovation sector locates in the same country.

Here, we discuss some welfare and policy implications of our model. In our model, decline in trade costs improves welfare of the whole economy, if growth rates are raised with decline in trade costs. However, it is possible that welfare of a country decreases with globalization (decline in trade costs), even if globalization fosters economic growth. With the decline in trade costs, location of firms changes: firms shift location from a country to the other country. Then welfare of a country which loses firms may decreases with globalization, while
globalization improves the welfare of a country which get firms. When trade costs is high, firms agglomerate to North with globalization. On the other hand, when trade costs are low, firms shift the location from North to South. Therefore, when trade costs are high, North assume to apply the policies which lowers trade costs, while South oppose to those policies. However, when trade costs are low, South applies the policy which promotes globalization, while North oppose those policies.

This discussion points out that there are conflicts between two countries: one country assumes to apply policies which lowers trade costs, such as Free Trade Agreement (FTA), and while the other country oppose to the policies. In the case that decline in trade costs raises the growth rates, coordination of the policies to lower trade costs improves the welfare of the whole economy. In that case, organization like WTO is necessary for the economy to achieve at the high growth rates. Our model, pointed out the above welfare and policy implication.

4 Conclusion

In this paper, we have constructed a model in which equilibrium wages in two countries differ. Differences in wage rates and market size generate particular patterns of growth and agglomeration in the economy. If the equilibrium wages in two countries are the same, the country with a large market always absorbs more firms than the other country. Therefore, when the equilibrium wages in two countries are the same, the innovation sector always locates in the large-market country, and growth rates are raised with agglomeration of manufacturing firms in that country. In this case, the share of manufacturing firms in the large-market country increases with a decline in trade costs.

However, if wages in the large-market country are higher than those in another country, higher wages lower the share of manufacturing firms in the large-market country. We show the relationship between the proportion of manufacturing firms and the reduction in trade costs. When trade costs are high, the share of manufacturing firms in the large-market country increases with a decline in trade costs, and we observe full agglomeration in the large-market country. However, when trade costs become low, full agglomeration in the large-market country is broken, and some manufacturing firms locate in the small-market country. We show that, finally, all manufacturing firms agglomerate in the small-market country when trade costs are very low. In this process, the innovation sector shifts its location from the large-market and high-wage country to the small-market and low-wage country, and economic growth rates first rise, then fall, and finally rise again. We studied the effects of reduction in trade costs on the location of manufacturing firms and economic growth rates. By introducing the difference of wages in two countries, the results become richer, and the equilibrium in which manufacturing firms agglomerate in a small-market country and the innovation sector locates in that country can be observed.
Appendix

1. The proof that the firm share \( s^* = s^*(\lambda, A, \phi) \) is an increasing function of \( A \)

We show that \( s^*(\lambda, A) \) is an increasing function of \( A \). Differentiating (18) with respect to \( A \), we obtain

\[
\frac{\partial s^*}{\partial A} = s^* \left[ \frac{(1 + \phi)(1 - \phi) \lambda + \phi^2}{A(1 + \phi)(1 - \phi) A - \phi(1 - \phi A)} + \frac{(1 + \phi^2 - 2\phi A)}{(1 + \phi A)(A - \phi)} \right] = \left[ \frac{(1 + \phi)(1 - \phi) \lambda + \phi^2 - (1 + \phi^2 - 2\phi A)s^*}{(1 + \phi A)(A - \phi)} \right].
\]

From an interior solution condition, the denominator of (22) satisfies \((1 + \phi A)(A - \phi) > 0\). Therefore, we check the sign of the numerator of this equation. (18) can be transformed as follows:

\[
(1 + \phi)(1 - \phi) \lambda + \phi^2 = \left(1 - \frac{\phi}{A} - \phi(A - \phi)\right)s^* + \frac{\phi}{A}
\]

Then, the numerator of (22) is

\[
(1 + \phi)(1 - \phi) \lambda + \phi^2 - (1 + \phi^2 - 2\phi A)s^* = \left(1 - \frac{\phi}{A} - \phi(A - \phi)\right)s^* + \phi \left(\frac{1}{A} - s^* + s^*A\right) > 0.
\]

In other words, \( \frac{\partial s^*}{\partial A} > 0 \) has been shown.

2. Proof of proposition 1

From (18), we can derive that

\[
\frac{\partial s^*}{\partial \phi} = \frac{A\lambda}{(1 - A\phi)^2} - \frac{A(1 - \lambda)}{(A - \phi)^2}
\]

Therefore,

\[
\frac{\partial s^*}{\partial \phi} \bigg|_{\phi=0} = A\lambda - A(1 - \lambda)/A^2.
\]
Here, we assume that \( \lambda \geq \frac{1}{1\lambda \lambda \lambda} \). In this case, \( \frac{\partial s^*}{\partial \phi} \big|_{\phi=0} = A\lambda - \frac{A(1-\lambda)}{\lambda \lambda \lambda} > 0 \).

In addition,

\[
\frac{\partial s^*}{\partial \phi} \big|_{\phi=A} = -\infty.
\]

Equation (23) shows that, when \( \frac{\partial s^*}{\partial \phi} = 0 \),

\[
(A^2 + \lambda - A^2\lambda)\phi^2 - (4A\lambda - 2A)\phi + A^2\lambda + \lambda - 1 = 0.
\]

This means that a function \( s^*(\phi) \) has, at most, two extreme values of \( \frac{\partial s^*}{\partial \phi} = 0 \).

Here, \( \frac{\partial s^*}{\partial \phi} \big|_{\phi=0} > 0 \) and \( \frac{\partial s^*}{\partial \phi} \big|_{\phi=\lambda} < 0 \) mean that there is one extreme value, \( \phi^* \), where \( \frac{\partial s^*}{\partial \phi} \big|_{\phi=\phi^*} = 0 \) in \( 0 \leq \phi \leq \lambda \) and, when \( 0 \leq \phi \leq \phi^* \), \( \frac{\partial s^*}{\partial \phi} > 0 \) and, when \( \phi^* < \phi \leq \lambda \).

When all firms agglomerate in country 1 at equilibrium, Equation \( s^*(\lambda, A, \phi) = 1 \) for \( \phi \) has two real number solutions. (18) can be transformed as

\[
(1 + \phi)(1 - \phi)\lambda = (1 - \phi A),
\]
\[
\lambda\phi^2 - \phi A + 1 - \lambda = 0.
\]

For this equation to have two real number solutions, the following must be satisfied:

\[
D \equiv A^2 - 4\lambda(1 - \lambda) \geq 0.
\]

That is,

\[
\lambda \geq \frac{1 + (1 - A^2)^{\frac{1}{2}}}{2}.
\]

In the case of \( \lambda \geq \frac{1 + (1 - A^2)^{\frac{1}{2}}}{2} \), \( s^*(\lambda, A, \phi) \geq 1 \) is satisfied, when \( A - (A^2 - 4\lambda(1 - \lambda))^{\frac{1}{2}} \equiv \phi_1 \leq \phi \leq \frac{A + (A^2 - 4\lambda(1 - \lambda))^{\frac{1}{2}}}{2\lambda} \equiv \phi_2 \).

When all firms agglomerate in country 2 at equilibrium, Equation \( s^*(\lambda, A, \phi) \leq 0 \) must be satisfied. This inequality leads us to \( \frac{1 - (1 - A^2\lambda(1 - \lambda))^{\frac{1}{2}}}{2A(1 - \lambda)} \equiv \phi_2 \leq \phi \leq \frac{1 - (1 - A^2\lambda(1 - \lambda))^{\frac{1}{2}}}{2A(1 - \lambda)} \equiv \phi_3 \).

Here, \( \frac{1 - (1 - A^2\lambda(1 - \lambda))^{\frac{1}{2}}}{2A(1 - \lambda)} \geq 1 \), Thus, all firms agglomerate in country 2 when \( \phi_3 \leq \phi \leq 1 \).

This discussion means that \( \phi_1 < \phi^* < \phi_2 < \phi_3 < A \). Then, in the case of \( \lambda \geq \frac{1}{1\lambda \lambda \lambda} \), \( \frac{\partial s^*}{\partial \phi} > 0 \), when \( 0 \leq \phi \leq \phi_1 \) and \( \frac{\partial s^*}{\partial \phi} < 0 \), when \( \phi_2 \leq \phi \leq \phi_3 \).

In the case in which the parameters satisfy the relation \( \lambda \geq \frac{1 + (1 - A^2)^{\frac{1}{2}}}{2} \), when \( \phi_1 \leq \phi \leq \phi_2 \), \( s^* = 1 \). On the other hand, when \( \phi_3 \leq \phi \leq 1 \), \( s^* = 0 \).
3. The condition for $\hat{s} - \frac{1}{1 + A^2} > 0$

Here,

$$\hat{s} - \frac{1}{1 + A^2} = \frac{\delta (A^\frac{1}{1+\sigma} - A^2) + A^{\frac{3-2\sigma}{1+\sigma}} - 1}{(1 + A^2)(1 + A^\frac{1}{1+\sigma})(1 - \delta)}.$$ 

Therefore, $\hat{s} - \frac{1}{1 + A^2} > 0$ means

$$\delta > \frac{1 - A^{\frac{3-2\sigma}{1+\sigma}}}{A^\frac{1}{1+\sigma} - A^2}.$$ 

References


Figure 1: The location of innovation sector
Figure 2: The relation between trade costs and share of manufacturing firms
Figure 3: The relation between trade costs and steady state growth rate