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Stochastic Origin of Scaling Laws in Productivity and Employment Dispersion

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Abstract

Labor and productivity play central roles in the aging population problem in all developed countries. The understanding of labor allocation among different productivity levels is required for policy issues, specifically, the dynamics of how workers are allocated and reallocated among sectors. We uncover an empirical fact that firm-level dispersions of output and employment satisfy certain scaling laws in their joint probability distributions, which closely relate to the dispersion of productivity. The empirical finding is widely observed in large databases including small and medium-sized firms in both Japan and European countries. We argue that a stochastic process generates a steady-state allocation of labor across firms of differing output and productivity, which results in the observed distributions of workers, productivity, and output.

Keywords: labor productivity, dispersion, scaling laws, and stochastic process.

JEL classification: C02; E20; O40

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I. Introduction

Labor and productivity play a central role in the problems of ageing population with low birth-rate in developed countries. Policy making should involve utilization of bounded labor forces and maintaining productivity at a certain level. Understanding of the dynamics — how workers are allocated and reallocated to sectors with differing productivity under changing demand — would be highly required as a basis of arguments for any policy making.

Empirically, productivity has dispersion among firms and sectors (see Shinohara (1955); Salter (1960); Yoshikawa (2000a) and references therein). The study of productivity dispersion is aided by the literature in labor economics (e.g. Davis et al. (1996); Albaek and Sorensen (1998); Mortensen (2003)). Mortensen (2003), for example, concluded as saying that the problem of wage dispersion, namely why are similar workers paid differently, reflects differences in employer productivity. Quoting from its afterword,

I have argued that wage dispersion of this kind reflects differences in employer productivity. More productive employers offer higher pay to attract and retain more workers. Workers flow from less to more productive employers in response to these pay differences, and both workers and employers invest search and recruiting efforts in that reallocation process. Exogenous turnover and job destruction on the one hand, and search friction on the other, prevent the labor market from ever attaining a state in which all workers are employed by the most productive firms. Instead, a continuous process of reallocation of workers from less to more productive employers interrupted by transitions to nonemployment induced by job destruction and other reasons for labor turnover generates a steady-state allocation of labor across firms of differing productivity.

It should be noted that the dynamical processes of allocation of labor across firms of differing productivity are essentially of stochastic nature.

Theoretically, since the productivity dispersion relates to the notion of equilibrium in economics, it concerns the foundation of economic theory as pointed out in the pioneering works (Yoshikawa, 2000a,b; Aoki and Yoshikawa, 2007). They claim the necessity of a concept of equilibrium as *distribution* in conformity with the above quote on the concept of a steady-state allocation of labor across firms of differing productivity.

This working paper focuses on this point on the steady-state distribution as equilibrium. To be specific, the paper studies firm-level dispersions of productivity, employment and output by using large databases including small and medium-sized firms in Japan and in European countries. First, we uncover an empirical and universal fact that firm-level dispersions of output and employment satisfy what we call scaling laws in their joint probability distributions. This fact has a strong implication and, indeed, it follows as a mathematical consequence that the marginal

and joint distributions are determined for their functional forms. Second, we seek for the stochastic origin of these properties arguing that one can understand them by a stochastic process generating a steady-state allocation of labor across firms of differing output and productivity under a demand constraint.

In Section II, we describe our databases of Japan and of Europe briefly in Section II.A. We show marginal and joint distributions of productivity, output and labor in Section II.B. Scaling laws in the joint distributions of output and labor are uncovered in Section II.C, in which we prove mathematical consequences from the scaling relations. In Section III, we argue about a possible stochastic origin of our findings based on a jump Markov process. We summarize in Section IV.

II. Empirical Findings

A. Databases

We employ two databases, which cover small and medium-sized firms as well as large ones, of Japan and of a few European countries. In the both of them, the basic variables such as number of employees, operating revenue, total assets, sales and so forth are available in addition to industrial sectors and other attributes at the level of firms.

The Japanese database is the Credit Risk Database (CRD), a compiled set of financial data and default information that are mainly focused on small and medium-sized firms and are based on the credit information provided by the domestic credit guarantee corporations and financial institutions. The database includes roughly a million firms and fifteen million workers in Japan, the coverage of which would be satisfactory for our purpose (see Aoyama et al. (2009) for the more details). We select the manufacturing sectors (defined by the primary business sector of each firm according to the Japan Standard Industrial Classification).

The European database is the Bureau van Dijk's ORBIS, a collection of globally known datasets over the world, in which European countries are well covered for their descriptive and financial data. We utilized the database to select France, Germany, Italy and UK, which are relatively well covered with respect to the number of firms. Actually, for a firm to be included in each dataset, it must satisfy at least either of the criteria; operating revenue equal or greater than 1 million Euro, total assets equal or greater than 2 million Euro, or number of employees equal or greater than 15. We present results for France and industrial (non-financial) sectors in the Appendix, but remark that the others have similar results.

Labor productivity c of each firm is calculated by

$$c = \frac{Y}{L}, \quad (1)$$

where Y is the value added and L is the labor measured in number of workers. As a proxy of value added, we shall use the output of firms measured by business

sales for the CRD and operating revenue (turnover) for the ORBIS databases. In a preliminary analysis, we checked that the results are quite robust and independent of a particular choice of proximate variables as the output Y . (See our previous work (Aoyama et al., 2009) for the calculation of added values).

B. *Distributions of Productivity, Workers and Outputs*

To quantify how workers are allocated among differing output and productivity, we shall use the distributions of the variables c , L and Y . Let us introduce the following probability distribution functions (PDFs). Namely

$$P_{YL}(Y, L) = P_{Y|L}(Y|L)P_L(L) = P_{L|Y}(L|Y)P_Y(Y) , \quad (2)$$

where P_{YL} is the joint PDF; $P_{Y|L}$ and $P_{L|Y}$ are the conditional PDFs; P_L and P_Y are the marginal PDFs. For an arbitrary function $f(\cdot)$, its conditional average is defined by

$$E(f(Y)|L) := \int_0^\infty f(Y)P_{Y|L}(Y|L)dY , \quad (3)$$

which is the average conditioned on L . Similarly for $E(f(L)|Y)$.

The marginal PDFs have heavy-tails as shown in Fig. 1 for the Japanese data (see also Fig. 5 in the Appendix). Therefore, it is convenient to use the logarithms of the variables for the purpose of statistical analysis. For notational simplicity, let us denote them as

$$y = \log Y/Y_0 , \quad \ell = \log L/L_0 , \quad (4)$$

where Y_0 and L_0 are arbitrary scales.

Let us now proceed and examine the joint PDFs. Fig. 2 (a) shows the scatter plot for (ℓ, y) , in which the conditional average $E(y|\ell) = E(\ln Y|L)$ is shown by a line. Obviously, there exists a certain range of L for which the relation:

$$E(y|\ell) = \alpha\ell + \text{const.}, \quad (5)$$

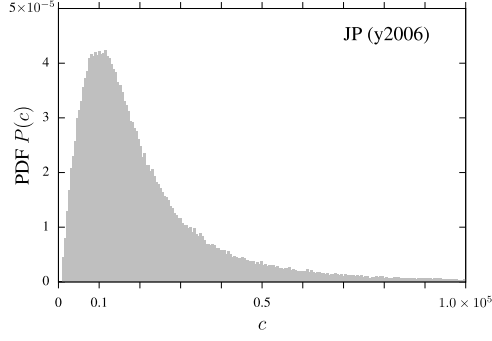
holds where α is a constant. See Aoyama et al. (2010) for a kernel-based nonparametric methods for a statistical estimate of the above relation. The estimated value is $\alpha = 1.037(\pm 0.003)$.

Similarly, Fig. 2 (b) shows the scatter plot for (y, ℓ) and the conditional average $E(\ell|y)$. For a certain range of Y , one has

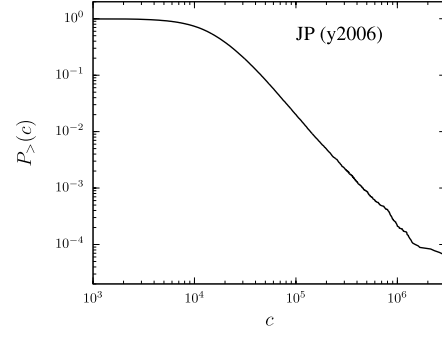
$$E(\ell|y) = \beta y + \text{const.}, \quad (6)$$

with a constant β , which is estimated as $\beta = 0.655(\pm 0.002)$.

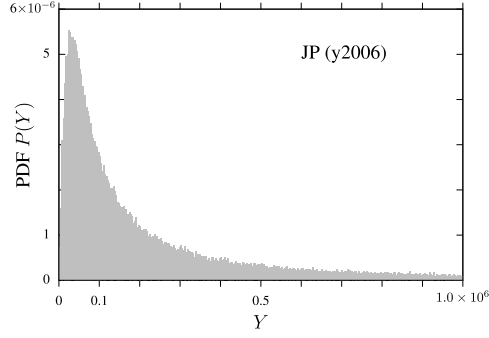
See also Fig. 6 in the Appendix for the validity in the case of the other dataset.



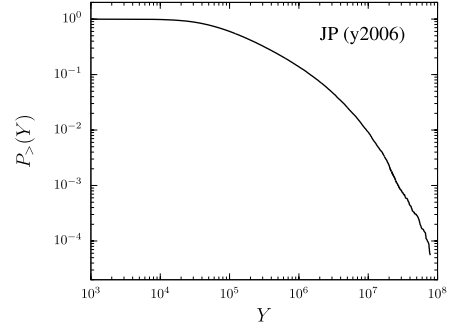
(a) PDF for productivity $c = Y/L$



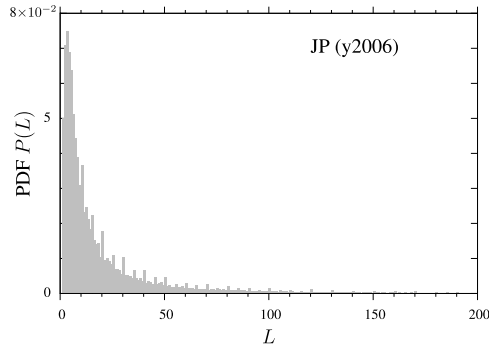
(b) Cumulative PDF for c



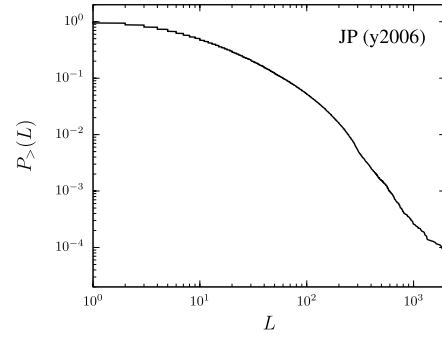
(c) PDF for output Y



(d) Cumulative PDF for Y



(e) PDF for output L



(f) Cumulative PDF for L

Figure 1: Data: CRD in the year 2006 (Japan). Y is output measured by sales (in thousand yen) and L is the number of workers.

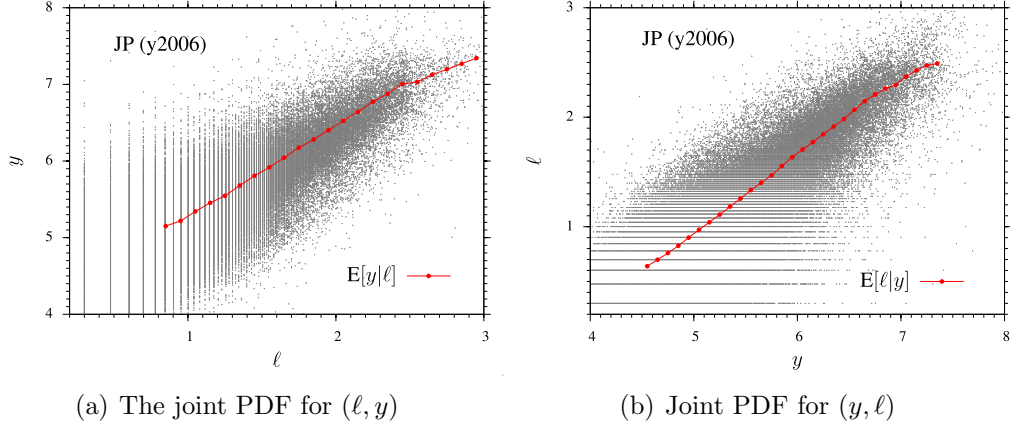


Figure 2: Data: CRD in the year 2006 (Japan). The lines are the conditional averages, $E(y|\ell)$ and $E(\ell|y)$.

C. Scaling relations and consequences

Now we consider the relations in (5) and (6). They mean that a larger number of workers are employed in firms with higher levels of output, in the *average* sense, in nearly the entire range of variables.

Furthermore, we find that these relations are consequences from two *scaling relations* for the conditional PDFs, $P_{Y|L}(Y|L)$ and $P_{L|Y}(L|Y)$. To see this fact, we depict in Fig. 3 (a) the conditional PDF, $P_{Y|L}(Y|L)$, with the conditioning values of L are chosen at a logarithmically equal interval corresponding to a certain range of L in terms of histograms. By using the values of α estimated above, we find that the conditional PDF obeys a scaling relation:

$$P_{Y|L}(Y|L) = \left(\frac{L}{L_0}\right)^{-\alpha} \Phi_Y(Y_{\text{scaled}}) , \quad (7)$$

where $Y_{\text{scaled}} := (L/L_0)^{-\alpha}Y$ and $\Phi_Y(\cdot)$ is a scaling function, as shown by the fact that the PDFs $P_{Y|L}(Y|L)$ for different values of L fall onto a curve depicted in Fig. 3 (b). It is straightforward to show that (5) follows from (7).

Similarly, we have another scaling relation for

$$P_{L|Y}(L|Y) = \left(\frac{Y}{Y_0}\right)^{-\beta} \Phi_L(L_{\text{scaled}}) , \quad (8)$$

where $L_{\text{scaled}} := (Y/Y_0)^{-\beta}L$ and $\Phi_L(\cdot)$ is a scaling function, as shown by Fig. 4 (a) and (b). And also Eq. (6) immediately follows from Eq. (8). See also Fig. 7 in the Appendix for the validity in the case of the other dataset; so the results are universal.

Surprisingly, the two scaling laws, Eqs. (7) and (8), which we collectively call *the double scaling law* (DSL) have strong consequences to the function form of the joint PDF as we shall show in the following.

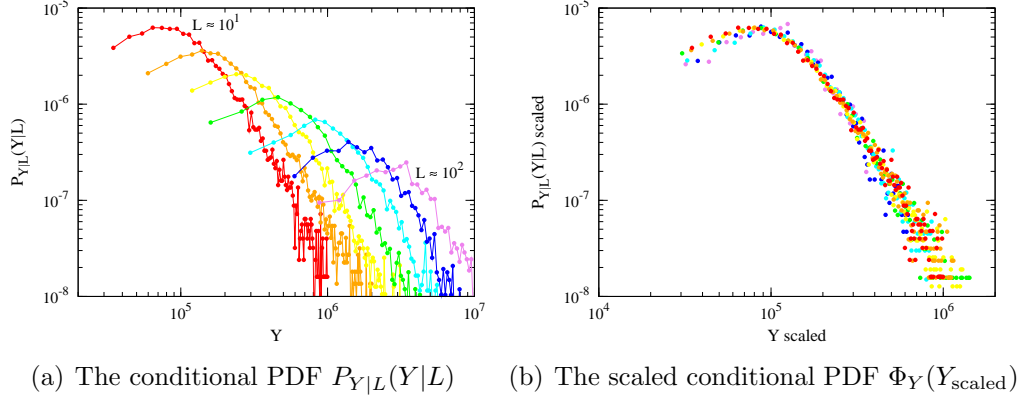


Figure 3: Data: CRD in the year 2006 (Japan)

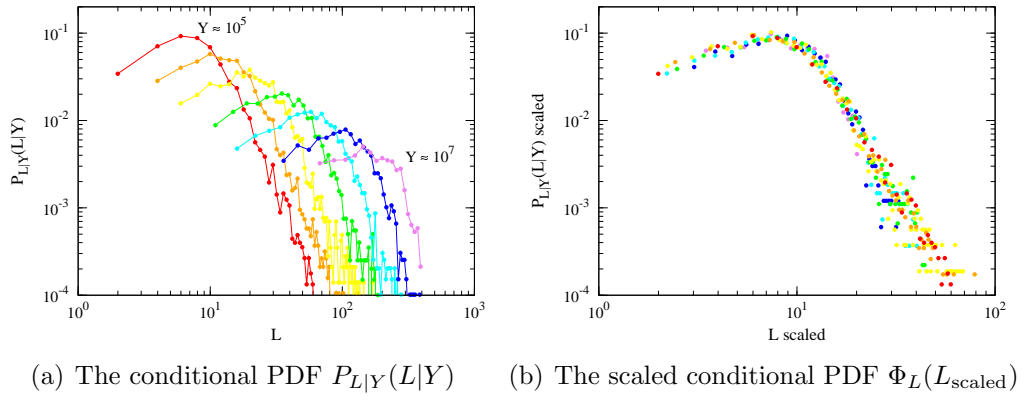


Figure 4: Data: CRD in the year 2006 (Japan)

Let us choose the reference scales Y_0 and L_0 to be within the region of the (Y, L) -plane where DSL is valid. Then by substituting $Y = Y_0, L = L_0$ into Eqs. (2), (7) and (8), We obtain the marginal PDFs, P_Y and P_L in terms of the invariant functions, Φ_Y and Φ_L :

$$P_L(L) = \left(\frac{L}{L_0}\right)^\alpha \frac{\Phi_L(L)}{\Phi_L(L_0)} \frac{\Phi_Y(Y_0)}{\Phi_Y((L/L_0)^{-\alpha}Y_0)} P_L(L_0) , \quad (9)$$

$$P_Y(Y) = \left(\frac{Y}{Y_0}\right)^\beta \frac{\Phi_Y(Y)}{\Phi_Y(Y_0)} \frac{\Phi_L(L_0)}{\Phi_L((Y/Y_0)^{-\beta}L_0)} P_Y(Y_0) . \quad (10)$$

From Eqs. (7) and (8) and the above, we arrive at the following equation for the Φ s:

$$\frac{\Phi_L((Y/Y_0)^{-\beta}L)}{\Phi_L((Y/Y_0)^{-\beta}L_0)} \frac{\Phi_L(L_0)}{\Phi_L(L)} = \frac{\Phi_Y((L/L_0)^{-\alpha}Y)}{\Phi_Y((L/L_0)^{-\alpha}Y_0)} \frac{\Phi_Y(Y_0)}{\Phi_Y(Y)} . \quad (11)$$

This equation puts strong constraints the form of Φ 's. We have converted the above to differential equations and have derived *complete solutions* of Eq. (11). Depending on whether $\alpha\beta = 1$ or not, the solution is qualitatively different, which we shall explain below.

When $\alpha\beta = 1$, we find the following relation between Φ 's is necessary and sufficient for Eq. (11):

$$\Phi_L(L) = \Phi_Y((L/L_0)^{-\alpha}Y_0) \left(\frac{L}{L_0}\right)^a \frac{\Phi_L(L_0)}{\Phi_Y(Y_0)} . \quad (12)$$

In other words, we have one arbitrary function in the solution. In this case, Eqs. (9) and (10) implies that

$$P_L(L) = \left(\frac{L}{L_0}\right)^{-\mu_L-1} P_L(L_0) , \quad (13)$$

$$P_Y(Y) = \left(\frac{Y}{Y_0}\right)^{-\mu_Y-1} P_Y(Y_0) , \quad (14)$$

with

$$\alpha = \frac{\mu_L}{\mu_Y}, \quad \beta = \frac{\mu_Y}{\mu_L}, \quad a = -\frac{\mu_L + \mu_Y + \mu_L\mu_Y}{\mu_Y} . \quad (15)$$

This result is straightforward to understand: Due to $\alpha\beta = 1$, we have

$$L_{\text{scaled}}^{-\alpha} \propto Y L^{-\alpha} \propto Y_{\text{scaled}} . \quad (16)$$

Therefore, an arbitrary function of Y_{scaled} is a function of L_{scaled} as far as dependence on Y and L is concerned. This is why an arbitrary function is left in the solution. Also the marginal PDFs in Eqs. (13) and (14) results from the relation (2).

For $\alpha\beta \neq 1$, we obtain,

$$\Phi_Y(Y) = e^{-\beta p y^2 + q y} \Phi_Y(Y_0) , \quad (17)$$

$$\Phi_L(L) = e^{-\alpha p \ell^2 + s \ell} \Phi_L(L_0) , \quad (18)$$

$$P_L(L) = e^{-\alpha(1-\alpha\beta)p\ell^2 + (s+(q+1)\alpha)\ell} P_L(L_0) . \quad (19)$$

$$P_Y(Y) = e^{-\beta(1-\alpha\beta)p y^2 + (q+(s+1)\beta)y} P_Y(Y_0) . \quad (20)$$

The joint PDF is given by the following:

$$P_{YL}(Y, L) = e^{-\alpha p \ell^2 + 2\alpha\beta p \ell y - \beta p y^2 + s \ell + q y} P_{YL}(Y_0, L_0) . \quad (21)$$

We find that in the limit $\alpha\beta \rightarrow 1$ we obtain the power laws for $P_L(L)$ and $P_Y(Y)$, which is consistent with the results (13) and (14) above.

One can show that the above theoretical results are in good agreement with the empirical data. See Aoyama et al. (2010) for it.

In summary, for the generic case of $\alpha\beta \neq 1$, one has *lognormal* distributions for the marginal and joint PDFs. This mathematical consequence from the DSL provides a strong constraint in the underlying dynamics. We shall proceed to consider the stochastic origin of the lognormal distributions and the scaling laws.

III. Stochastic model

We suppose that the labor and production factors cannot be reallocated instantaneously to the sector of highest productivity. Instead, at each moment of time, there exist dispersions or distributions of labor, output and productivity as observed empirically. We employ a jump Markov process to describe the reallocation process in a stochastic way. The purpose of this section is merely a sketch of our model.

Let us consider a sector in which firms increase or decrease their outputs. The firms are subject to relative demands which are continually shocked by events such as changes in the tastes of consumers. The idiosyncratic shocks are the origin of stochastic variation in the output Y_i of firm i . Additionally, let us assume that the total output $Y \equiv \sum_i Y_i$ is constrained $Y = D$ by the total demand D to understand the basic nature of the model. It would be easy to generalize it to the case of fluctuating demand.

In a time interval dt , the firm i changes its output Y_i by a small amount of unit, which is assumed to be unity without loss of generality. We denote by x the output measured in the unit just mentioned, and by x_0 the maximum value determined by the constraint of total demand. During the interval, the firm can grow or shrink in its output by a unit with probability $g_x dt$ or $h_x dt$ respectively. In other words, g_x is the transition rate for the process $x \rightarrow x + 1$, and h_x for $x \rightarrow x - 1$. The shrink of a firm with $x = 1$ is interpreted as the firm's exit. Additionally, we assume that a small firm with $x = 1$ can enter with probability νdt .

We denote the number of firms with output x by $n_x(t)$. The stochastic process of transition, entry and exit can be then described by the master equation:

$$\frac{\partial n_x}{\partial t} = g_{x-1} n_{x-1} + h_{x+1} n_{x+1} - (h_x + g_x) n_x \quad (22)$$

for $1 < x < x_0$, and

$$\frac{\partial n_1}{\partial t} = \nu + h_2 n_2 - (h_1 + g_1) n_1 \quad (23)$$

$$\frac{\partial n_{x_0}}{\partial t} = g_{x_0-1} n_{x_0-1} + h_{x_0+1} n_{x_0+1} . \quad (24)$$

We are interested in the stationary state such that $\partial_t n_x = 0$. It can be readily obtained by standard methods (see Ijiri and Simon (1977); Steindl (1965) for example). Noting that the boundary condition is give by $n_x = 0$ for $x \leq 0$, one can easily show that

$$n_x = \frac{\nu}{h_1} \prod_{k=1}^{x-1} \frac{g_{x-k}}{h_{x-k+1}} . \quad (25)$$

Let us now specify the model as follows. Consider a firm i among K firms and the other $K - 1$ firms in a mean-field theory.

1. With probability $1 - \mu$, a vacancy-occupation process takes place. That is, pick two units of output randomly and suppose that they belong to firms i and j . By equal chance, the firm i grows while the firm j shrinks, or j grows while i shrinks. This process describes a decrease of the output of a firm, or vacancy, is immediately occupied by an increase of the output of another firm. The probability that a particular firm is selected is proportional to the share of the firm.
2. With probability μ , an immigration from outside the sector takes place. That is, choose a unit of output randomly and suppose that it belongs to a firm i . Replace the unit of output by a new firm.

These processes can be represented by a set of transition rates:

$$g_x \propto (1 - \mu) \frac{x}{x_0} \frac{x_0 - x}{x_0 - 1} + \frac{\mu}{K} \left(1 - \frac{x}{x_0} \right) , \quad (26)$$

$$r_x \propto (1 - \mu) \frac{x}{x_0} \frac{x_0 - x}{x_0 - 1} + \frac{\mu}{K} (K - 1) \frac{x}{x_0} . \quad (27)$$

What is essentially important in these expressions is the transition rates g_x and r_x are quadratic functions of x . The stationary solution (25) with the transition rates (26) and (27) is basically equivalent to the well-known Ewens distribution (see Aoki and Yoshikawa (2007) for its usage in various contexts).

The present model can be understood easily with the help of an analogy with an ecological system. Imagine that n_x is the number of species with the number of individuals x . The demand constraint corresponds to a resource constraint in the ecosystem, where species grow or shrink their individuals. A vacancy is immediately occupied by an individual of possibly another species. The probability that a particular species is selected is proportional to the size of the species. In addition, there exists a chance of immigration of new species from outside the system. Thus our model is a generic birth-death process with immigration. It should be mentioned that the well-known Simon's model (Ijiri and Simon, 1977) is a pure birth process which generates a power law distribution. In contrast, the present model is known to generate a lognormal distribution for a certain range of parameter of immigration. See Hubbel (2006) for example in the context of ecological system. In fact, using the transition rates (26) and (27) in the stationary state (25), one can show that the distribution can be well approximated by a lognormal distribution.

Moreover, workers employed in the same firm are subject to the same aggregate shocks, which tend to increase or decrease the number of workers. Here the aggregate shocks mean the demand shocks that we described in the above. Thus our model can be naturally extended to include the dynamics of workers by postulating that the transition $L \rightarrow L + 1$ at a moment is driven by the increase of output at a previous time corresponding to the growth of demand, $Y \rightarrow Y + \Delta Y$ ($\Delta Y > 0$). And similarly $L \rightarrow L - 1$ by the decrease of output or the shrink of demand, $Y \rightarrow Y - \Delta Y$ at a previous time. In other words, the allocation and reallocation dynamics of workers are subordinated to the dynamics of outputs of firms. The joint distributions of L and Y are therefore well approximated by the distributions of lognormal types. It remains to link the mathematical properties found in the preceding section with the present model and parameters in it. This important point is left as a future work.

IV. Summary

By employing the large databases including small and medium-sized firms in Japan as well as in Europe, we uncover an empirical fact that firm-level dispersions of output and employment satisfy certain scaling laws in their joint probability distributions, which we call double scaling law. The empirical finding seems to be universally observed. Surprisingly, it follows from the double scaling law that the marginal and joint distributions for output and labor should be lognormal in a generic case, and be of power law types in a special case.

As we had shown in our previous paper, these non-trivial properties lead that (1) the fact that the power law in the PDF $P_c(c)$ holds for a wide range of productivity; in addition, the origin of the value $\mu_c \simeq 2 \sim 3$, which is larger than the Zipf law $\mu = 1$ for firm-size including for the number of workers and the output), and that (2) $E[w|c]$ is an *increasing* function of c in a region corresponding to small productivity (Fujiwara and Aoyama, 2010).

We also sketch a stochastic process, mathematically a jump Markov process, in which workers are allocated to firms of differing output and productivity, interrupted by transitions to unemployment. We propose that the stochastic process generates a steady-state allocation of labor across firms of differing output and productivity resulting in the distributions of workers, productivity and output, although a full comparison of the model with the obtained empirical facts should be done in a future work.

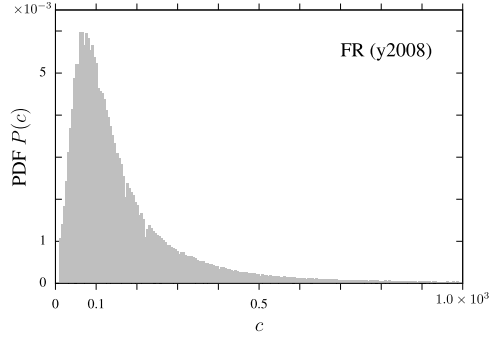
Our empirical finding of the scaling relations might be considered as a generalization of *production function* in the following sense. The concept of production function is a functional relation between output Y , labor input L and capital input K , where one usually assumes homogeneous production function, i.e. assumption of how output scales under a scaling of K and L . Our discovery is a *distributional* generalization of production function, which takes into account conditional distributions rather than conditional averages. This insight can potentially lead to a new perspective to look at distributions of productivity, labor, capital and output.

Acknowledgements

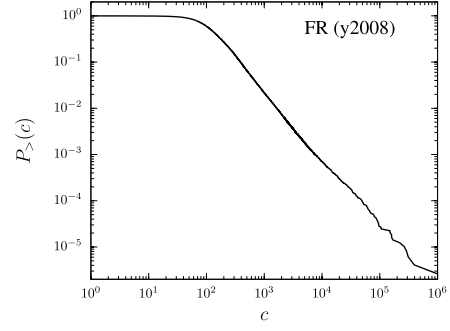
We are grateful for helpful comments and suggestions by Hiroshi Yoshikawa, Mauro Gallegati and the RIETI project participants, and for discussions with Yuichi Ikeda, Hiroshi Iyetomi and Wataru Souma. We thank the Credit Risk Database and the Joint Usage and Research Center, Institute of Economic Research, Kyoto University for the data used in this paper. This work is supported in part by the Program for Promoting Methodological Innovation in Humanities and Social Sciences by Cross-Disciplinary Fusing of the Japan Society for the Promotion of Science.

Appendix A. European data

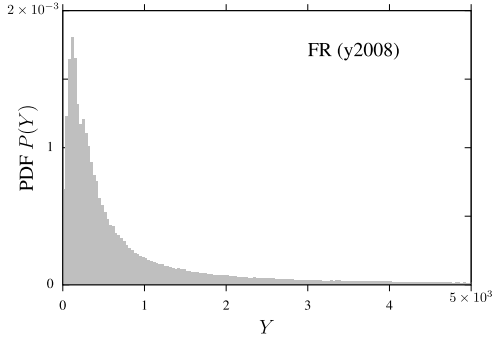
In this appendix, we summarize the results for the European database (ORBIS), specifically France and industrial (non-financial) sectors. Since the results are parallel to those obtained for the Japanese data (CRD), let us collect the plots in the following, which are referred to in the main text.



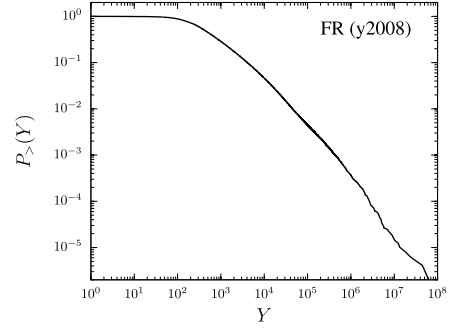
(a) PDF for productivity $c = Y/L$



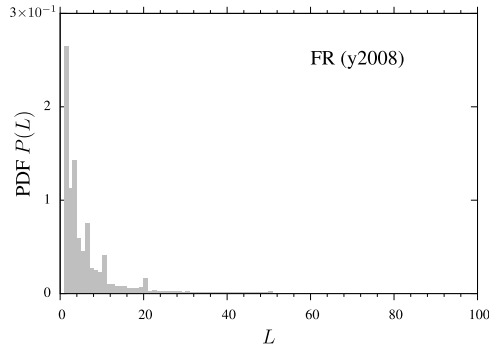
(b) Cumulative PDF for c



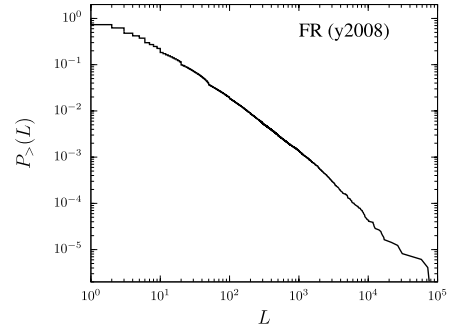
(c) PDF for output Y



(d) Cumulative PDF for Y



(e) PDF for output L



(f) Cumulative PDF for L

Figure 5: Data: ORBIS in the year 2009 (France)

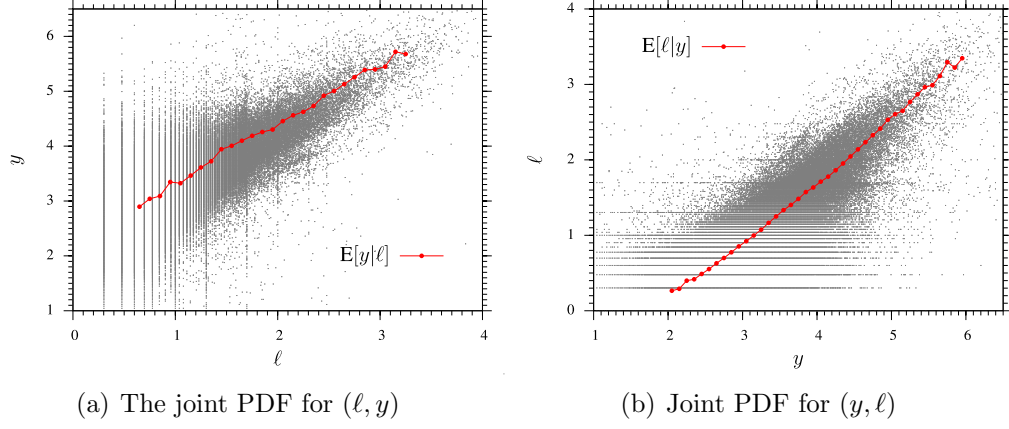


Figure 6: Data: ORBIS in the year 2009 (France)

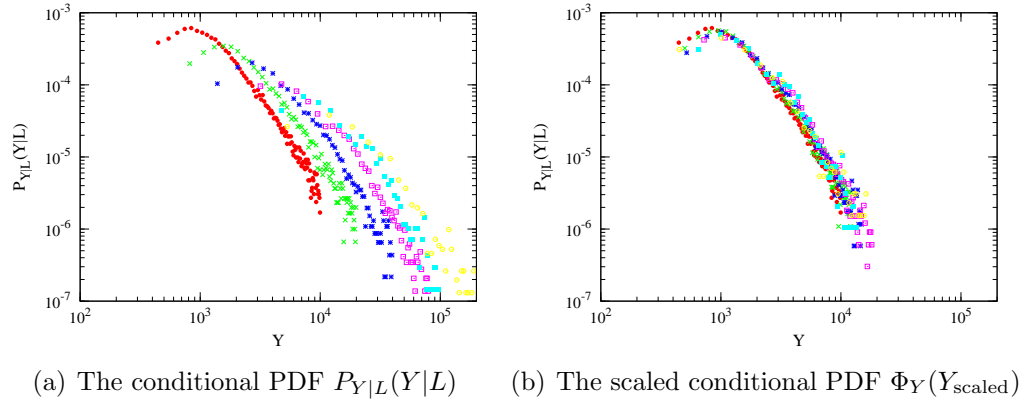


Figure 7: Data: ORBIS in the year 2009 (France)

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