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Abstract

The Japanese labor market displays U-shaped unemployment and separation rates, and declining job-finding rates as workers age. Traditional infinite horizon search models of the labor market cannot account for such patterns. We develop a life-cycle search and matching model that features random match quality and incorporates elements capturing several main characteristics of the Japanese labor market. We show that the model, calibrated for Japan, replicates the life-cycle properties of the data. Our model, following an empirically plausible productivity drop, produces changes in the steady state levels of the unemployment and finding rates similar in magnitude to those observed in Japan since the 1980s.

Keywords: search and matching, life-cycle, overlapping generation, match quality, Japan, and productivity decline.
JEL Classification: E24; J63; J64

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1 Introduction

Workers experience very different labor market outcomes at different stages of their working lives. In many countries, young workers suffer high rates of unemployment and job separation, which is sometimes referred to as a youth unemployment problem. Workers who become unemployed close to the retirement age, on the other hand, often face greater difficulties in finding a job than their younger peers. Many of these conditions are also present in the Japanese labor market. However, while the U.S. data, for example, displays declining levels in unemployment, job-finding and job-separation rates with age, in Japan the unemployment and separation rates are U-shaped. Such patterns may reflect idiosyncrasies of the so-called Japanese employment system.

The main objective of this paper is to build a life-cycle model that captures the main characteristics of Japan's labor market. In particular, we develop a life-cycle labor search and matching model that incorporates some of the key institutional features in Japan. We subsequently calibrate the model to match the long-run empirical evidence, and compare its predictions of three key variables, the job-finding, separation, and unemployment rates, with the data. We then study model's behavior under a permanent decline in productivity, and compare its numerical implications to the labor market change that took place in Japan from the 1990s to the 2000s.

While it is often argued that the Japanese labor market is in many respects different from those of Europe or the U.S., there are clear similarities in the life-cycle patterns of some variables that define workers' labor market experiences. As shown in Esteban-Pretel and Fujimoto (2010), unemployment, job separation, and job-finding rates in the U.S. are all highest for young workers and decline with age. This is also true in Japan, but in the Japanese case, unemployment and separation rates increase after the age of 50, although they never reach the level of the youngest age group. Hence, while in the U.S. these variables basically decline monotonically with age, unemployment and separation rates in Japan are U-shaped. Moreover, the levels of these variables are generally lower in Japan compared to the U.S. Part of the difference in the shape of age profiles and the levels of these variables may be due to particularities in the Japanese labor market, such as severe restrictions to dismiss workers, widespread adoption of a mandatory retirement system, and a universal, highly organized job search during the last years of school.

In order to account for these features in the Japanese labor market, we build a model in the style of the recent life-cycle search and matching models such as Cheron, Hairault, and Langot (2008) and Hahn (2009), which differ from the textbook models[1] in assuming

[1]See, e.g., Pissarides (2000).

the finite life of workers. We further introduce several innovations to quantitatively explain general and Japan-specific features of the data. As in Esteban-Pretel and Fujimoto (2010), we incorporate random match quality, which is not necessarily known at match formation but is revealed after the first period of employment. As shown in Esteban-Pretel and Fujimoto (2010), the uncertainty in match quality delivers a declining separation rate as workers age, which the stylized model of Cheron, Hairault, and Langot (2008) does not obtain. We introduce other elements to capture some of the Japanese market idiosyncrasies: in particular, we assume that workers start searching for jobs before entry into the labor market, that it is costly to dismiss workers, and that that cost declines after the mandatory retirement age.

The model reproduces the empirical age profiles of the variables of interest well. In particular, our model generates the U-shaped unemployment and separation rates, as well as the declining job-finding rate, through the following mechanisms. First, the decline of the job-finding rate with age can be understood as follows: since we assume that workers of all ages are pooled into a single labor market, they all face the same probability of being matched with a firm. Age variations in the job-finding rate are therefore explained by differences in the probability that a match becomes a productive job. Older workers have shorter employment horizons ahead of them, which reduces the expected value of a job and thus makes firms more selective in hiring these workers, reducing their job-finding rate. This circumstance, already present in Cheron, Hairault, and Langot (2008), is known as the *horizon effect*.

The horizon effect, combined with the assumption that the firing cost is lower after a certain age, corresponding to the mandatory retirement age in Japan, yields a rise in the model separation rate during the last years in the labor market. However, the horizon effect alone generates an upward-sloping separation rate; this contradicts the empirical age profile, for which the separation rate is declining up to the prime age.

In our model, the existence of random match quality reverses this counterfactual prediction of the stylized model. This is through the destruction of many matches that are revealed as being of low quality, which occurs more frequently for younger workers for several reasons. First, young workers are more likely to find themselves in the first period of employment relationships, where the match quality may still be unknown. We assume that the probability of immediate match quality revelation is lower for younger workers, who have less experience in the labor market. Finally, since younger workers have a longer horizon before them, they have stronger incentives to leave a match that is revealed to be of low quality, and to seek better options. Overall, the horizon effect, the firing cost, and the match quality produce in the model the U-shaped separation rate observed in the data.

Finally, our model yields a U-shaped unemployment rate across ages close to that in the data. This is because, in our model, many young workers enter the labor market without a job and face higher separation rates, which in fact dominates the effect of their higher job-finding rates. The higher separation rate and lower job-finding rate for older workers generate their higher unemployment rate.

We also explore the predictions of the model for a permanent decline in productivity. This is a particularly interesting experiment for the Japanese economy as, following the seminal work of Hayashi and Prescott (2002), it has been argued in the literature that productivity declines produced the so-called "lost decade" of the 1990s. We find that our model, following an empirically plausible decline in productivity, generates a decrease in the job-finding rate and an increase in the unemployment rate similar in magnitude to those observed in Japan between the 1990s and the 2000s. However, the model fails to deliver the rise in separation rate seen during this period. The increase in unemployment and the decrease in the job-finding rate in the model are explained by the drop in profits, which follows the decline in productivity and reduces the incentives for firms to post vacancies. However, such a decline in the job-finding rate also reduces the outside option values for workers. This effect, together with the presence of a firing cost, makes the separation rate largely insensitive to declines in productivity.

Our paper is related to two strands of literature. First, it expands the growing literature that embeds a life-cycle or overlapping generations structure into labor search and matching models. This area was initiated by Pissarides (1992), which uses a two-period random matching model to study the effects of skill loss on the persistence of unemployment over the business cycle. Recent models allow far more general specifications in which workers live for many periods (Cheron, Hairault, and Langot (2008)) or for a constant duration in continuous time (Hahn (2009)), enabling more realistic analyses over the life-cycle. Menzio, Telyukova, and Visschers (2010) develops a life-cycle directed search model and attempts to match key facts over worker life-cycle, but differs from our model in adopting long-term contracts and allowing the labor market to be segmented by age, through firms posting contracts that specify applicant age.

Second, our paper relates to the literature on match quality and learning. Jovanovic (1979, 1984) develop models of the labor market in which gradual learning of the match quality leads to a decline in separation probability and a rise in the average wage as tenure increases. More recently, Pries and Rogerson (2005) embeds the learning of match quality into a search and matching model in order to quantitatively explain the differences in worker turnover between Europe and the U.S.

In this paper, we build on Esteban-Pretel and Fujimoto (2010), which combines these two approaches to study the U.S. labor market, and introduce additional features that capture some important characteristics of the Japanese labor market. We also differ from Esteban-Pretel and Fujimoto (2010) in exploring the implications of a decline in aggregate productivity, and comparing these to recent experiences in Japan.

The remainder of the paper is organized as follows. Section 2 explains some important empirical life-cycle facts, as well as several institutional features of the Japanese labor market. Section 3 presents the model, and Section 4 shows its baseline simulation results. Section 5 studies the behavior of the model after a productivity decline, and Section 6 concludes and offers a summary of the paper.

2 Key Features of the Japanese Labor Market

In this section, we discuss the Japanese labor market from empirical and institutional standpoints. We first document the life-cycle properties of the unemployment, job-finding, and separation rates; we then describe a number of key institutional features that are included in the model.

2.1 Life-cycle Patterns in the Data

We explore the life-cycle patterns for three of the main variables in the Japanese labor market, namely unemployment, job separation, and job-finding rates. Figure 1 shows the average values of these variables from 1983 to 2008 for five-year age groups. The data displayed in these figures is constructed using the Labor Force Survey and borrowed from Esteban-Pretel, Nakajima, and Tanaka (2011). The transition rates are quarterly and are obtained using the same methodology as in Shimer (2007), but they do not include the out of the labor force status.[2] The quarterly transition rates shown in the figures are the probabilities that a worker moves from unemployment to employment or vice versa during a three-month period, and therefore take into account all possible paths between the two states. As we observe in Figure 1, the unemployment rate and the worker transition rates have a marked age pattern, indicating that the experiences of workers of different ages are far from homogeneous. Let us analyze the relevant data in more detail.

[2] Since the model used in this paper does not include an inactivity state, our definitions of the monthly job-finding and separation rates are as follows: $JFR_t = \frac{UE_t}{UE_t+UU_t}$ and $JSR_t = \frac{EU_t}{EU_t+EE_t}$, where XY_t for $X, Y \in \{E, U\}$ is the number of workers whose employment state is X in month $t-1$ and Y in t. Shimer (2007) calculates those rates as $JFR_t = \frac{UE_t}{UE_t+UI_t+UU_t}$ and $JSR_t = \frac{EU_t}{EU_t+EI_t+EE_t}$, where $X, Y \in \{E, U, I\}$, which includes an inactivity state.

Unemployment rate by age is shown in Figure 1a. The average for the sample period across all ages is 3.4 percent. The unemployment rate has a clear U-shape, with the highest value being 8.8 percent for the youngest age group (15 to 19). It declines as workers get older, reaching its lowest point for workers aged 45 to 49, at 2.2 percent, and increases again to 5.6 percent for the oldest age group in the sample (60 to 64).

The separation rate, whose pattern can be seen in Figure 1b, also displays a U-shape. The average for all groups is 1.3% per quarter. Separation is highest for workers aged 15 to 19, at 3.9 percent, and lowest for those workers in the 45 to 49 age group, 0.85 percent. For the oldest workers, those aged 60 to 64, the separation rate is slightly higher than that of their younger counterparts, with a level of 1.5 percent.

Finally, the job-finding rate, shown in Figure 1c, declines as workers age. The average quarterly finding rate for all workers is 36.1 percent. Young workers just graduated from junior high-school, high-school, or college (aged 15 to 19 and 20 to 24) have the highest probability of moving into a job in a given quarter, around 44 percent. The finding rate begins to decline thereafter, reaching its lowest point for workers in the 60 to 64 age group, with a level of 21.9 percent per quarter.

In summary, young workers have higher unemployment and separation rates, although also higher job-finding rates. Middle-aged workers have the lowest unemployment and separation rates, but lower finding rates than very young workers. The oldest workers are likewise in a difficult situation, with higher unemployment and separation rates than those workers 10 to 15 years younger, and simultaneously the lowest probability of reemployment should they find themselves unemployed.

2.2 Some Institutional Features

We now briefly review some of the key institutional features of the Japanese labor market, which we incorporate into our model in Section 3.

First, young workers typically spend considerable time searching for their first job while still in school. Hiring processes for university graduates, for instance, can begin as early as the third year of their studies.[3]

Secondly, there is broad consensus that Japanese firms generally provide more training to their employees, especially in comparison with their U.S. counterparts.[4] While limitations in internationally comparable training data have restricted the direct verification of

[3]See e.g., Rebick (2005).

[4]See, e.g., Hashimoto and Raisian (1985) and Mincer and Higuchi (1988), which relate such training to the greater importance of firm-specific human capital in Japan.

this viewpoint, some empirical studies have found supporting evidence.[5]

Third, firing regular workers who are hired for an indefinite period is difficult in Japan. This condition is not obvious from a written labor law. For example, the Labor Standards Law, Article 20, states that an employer must pay no less than 30 days of average wage when failing to provide the employee with a 30-day advance notice of dismissal. This may seem to imply that an employer can freely dismiss a worker by paying a month of average wages: in reality, dismissal is severely restricted by the doctrine of abusive dismissal (*kaiko ken ranyo houri*).[6]

Fourth, a mandatory retirement system is prevalent. In 2002, 88% of firms, and 95% of companies with more than 5000 employees, had a uniform mandatory retirement system.[7] The typical age of mandatory retirement was 55 in the 1970s, but 60 is currently most common. Often, workers who have reached the retirement age are re-employed by the same employer, but on fixed-term contracts.

3 Model

We now develop a search and matching model of the labor market to account for the empirical facts described above. In order to study life-cycle issues, we depart from the standard specification by assuming that workers live only for a finite number of periods. Another important feature of our model is that matches differ in their quality, which is revealed at the start of the matches only with a certain probability. We show that this uncertainty in match quality is important to account for the data. Furthermore, we include several features to capture specific characteristics of the Japanese labor market, such as young workers' search before entering the labor market, or the restrictions to dismiss workers. We now examine the model in greater detail.

3.1 Environment

Time is discrete and runs to infinity. There are two agents in the economy, firms and workers, who try to meet in the labor market and form one-to-one employment relationships. Firms are ex ante homogeneous, and post vacancies by paying flow cost $k > 0$. Workers are in the labor market for T periods, although they start searching for jobs while

[5]Krafcik (1990) reports that in the automobile industry, the average hours of training in Japanese plants and Japanese-owned transplants in the U.S. are substantially greater than that in U.S.-owned plants.

[6]This doctrine has been developed since the 1950s through jurisprudence, and was recently codified in Labor Contract Law of 2008, Article 16. According to the translation by the Japan Institute for Labour Policy and Training, it reads as "A dismissal shall, if it lacks objectively reasonable grounds and is not considered to be appropriate in general societal terms, be treated as an abuse of right and be invalid."

[7]See OECD (2004), Table 4.1.

in school, before entering the market. This assumption reflects the highly structured and institutionalized search for jobs in Japan by students in their last year of school, as a result of which many individuals enter the labor market with a job.[8] After the T-th period in the labor market, a worker retires and exits the model. This T-th period does not in general coincide with the mandatory retirement age explained in Section 2.2, since many workers in Japan continue to work beyond the mandatory retirement age. There is a continuum of workers of each age, and the total mass of workers for each age group is normalized to 1.

Search by firms and workers in the model is random, and the labor market is not segmented by age. This implies that workers of all ages compete for the same types of jobs. Firms and workers match according to a constant returns to scale matching function, $m(v,s) = vq(\theta)$, where v and s are, respectively, the number of vacancies and the number of searchers, and $\theta \equiv \frac{v}{s}$ is the market tightness. The matching function implies that $q(\theta)$ is the probability for a firm to match with a worker, or the vacancy filling rate for a vacant firm, and that $\theta q(\theta)$ is the job-finding rate for an unemployed worker.

Matches in the economy differ along several dimensions. First, they differ in the age of the worker, or the proximity to retirement. Second, they are different in the overall quality of the match. Each firm-worker pair has a specific match quality, $\mu \in \{\mu_1, \mu_2, \ldots, \mu_N\}$, which is constant throughout the duration of the match. With probability R_a, which may depend on age, the actual level of μ is revealed when the match is formed. With probability $1-R_a$, the match quality is unknown at the moment of the match, and is revealed after one period of employment.[9] The distribution of match quality is independent of age, such that $\Pr(\mu = \mu_n) = \pi_n$, where $\sum_{n=1}^{N} \pi_n = 1$. Finally, matches also differ in their productivity. The productivity of a job, ϵ, is idiosyncratic to each match and is i.i.d. over time and across matches. It is drawn every period, including the initial period of the match, from a distribution G with support $[\epsilon_{min}, \epsilon_{max}]$. The output of a productive match is the product of these two components, i.e. $\mu\epsilon$. In order to start production, the firm needs to pay a cost c, which may include expenses such as set-up costs or those related to training the worker.

Destruction of matches occurs in three forms. Matches are destroyed at an exogenous rate δ. They are also destroyed endogenously if, for a given age and match quality, the idiosyncratic productivity is so low that the match surplus is negative. Both exogenous and endogenous destruction may occur for a newly formed match; a match may be immediately destroyed after being formed. Furthermore, a match is destroyed when the worker turns

[8]We recognize that students' job search processes in Japan differ substantially with education level. This assumption attempts to capture, in a very simple way, a broad feature of such job searches by students.

[9]Since idiosyncratic productivity ϵ and output $\mu\epsilon$ are observed without noise, μ is revealed after one period of employment. It is possible to introduce more complicated scenarios of learning, but we adopt this simple learning process since our main emphasis is not on the theoretical aspects of learning.

age T and retires. When a match is destroyed endogenously, the firm needs to pay a cost F_a. Note that F_a may depend on age, which allows a reflection of the general difference in job security before and after the mandatory retirement age in Japan.

Finally, we assume that wages in the match are determined as the Nash solution to a bargaining problem, and that there is free entry of firms.

3.2 The Problem of the Agents

The Problem of the Firm

Firms post vacancies in the labor market, and if matched with a worker, decide whether to start producing or to remain as a vacant firm. The present discounted value of posting a vacancy, denoted by V, can be expressed as:

$$\begin{aligned} V &= -k + \beta \Bigg\{ q(\theta) \sum_{a=-3}^{-1} \gamma_a \beta^{-a} \left[R_1 \sum_{n=1}^{N} \pi_n \widetilde{J}^0_{1,n} + (1 - R_1) \widetilde{J}^0_1 \right] \\ & q(\theta) \sum_{a=0}^{T-1} \gamma_a \left[R_{a+1} \sum_{n=1}^{N} \pi_n \widetilde{J}^0_{a+1,n} + (1 - R_{a+1}) \widetilde{J}^0_{a+1} \right] + (1 - q(\theta)) V \Bigg\}, \end{aligned} \tag{1}$$

where

$$\widetilde{J}^0_{a+1,n} = (1-\delta) \int_{\epsilon_{min}}^{\epsilon_{max}} \max\left\{ J^0_{a+1,n}(\epsilon'), V \right\} dG(\epsilon') + \delta V,$$

and

$$\widetilde{J}^0_{a+1} = (1-\delta) \int_{\epsilon_{min}}^{\epsilon_{max}} \max\left\{ J^0_{a+1}(\epsilon'), V \right\} dG(\epsilon') + \delta V.$$

The value V is composed of the flow cost of posting a vacancy, k, and the continuation value. A firm discounts the future at rate $\beta \in (0, 1)$, and is matched with a worker with probability $q(\theta)$. If it is matched with a worker in its last year of school (aged -3 to 0, since the model is quarterly), the firm stops searching until the worker turns age 1, and receives the expected discounted value of having matched with such worker. If the worker is already in the labor market (aged 1 to N), the firm receives the corresponding expected value. If the firm does not find a worker, it receives the value of vacancy.

The firm's expected value of being matched depends on the age of the worker, where γ_a is the fraction of individuals of age a in the pool of workers searching for a job,[10] and whether the quality of the match is revealed before starting production (which occurs with probability R_a for a worker of age a) or not. The revealed match quality equals μ_n with

[10] In particular, $\gamma_a = \frac{s_a}{\sum_{a=-3}^{T-1} s_a}$, where s_a is the number of searchers of age a, and the number of searchers is the number of students without a job offer plus the number of unemployed workers for age larger than 0. Formally, $s_{-3} = 1$, $s_a = [1 - \theta q(\theta)] s_{a-1}$ for $a \in \{-2, ..., 0\}$, and $s_a = u_a$ for $a \in \{1, ..., T-1\}$.

probability π_n.

We denote by $\widetilde{J}^0_{a,n}$ the expected continuation value for a firm of a newly filled job with known match quality μ_n with a worker of age a. This value, if the match is not exogenously destroyed (which occurs at rate δ), is composed by the choice between the value of being a vacancy and that of being a newly filled firm with a worker of age a, known match quality μ_n and idiosyncratic productivity ϵ, that is, $J^0_{a,n}(\epsilon)$. If the match is exogenously destroyed, the firm receives the value of being a vacancy.

We can similarly define $\widetilde{J}^0_a$ as the expected continuation value for a firm, before knowing the match quality, of a newly filled job with a worker of age a. In this expression, $J^0_a(\epsilon)$ represents the value for a firm of a newly filled job, with unknown match quality, with a worker of age a and idiosyncratic productivity ϵ.

The expression for the value for a firm of an existing match with a worker of age a, match quality μ_n, and idiosyncratic productivity ϵ, is:

$$J_{a,n}(\epsilon) = \mu_n \epsilon - w_{a,n}(\epsilon) + \beta \left[\mathbb{I}_{a<T} \widetilde{J}_{a+1,n} + (1 - \mathbb{I}_{a<T}) V \right], \tag{2}$$

where

$$\widetilde{J}_{a+1,n} = (1-\delta) \int_{\epsilon_{min}}^{\epsilon_{max}} \max \left\{ J_{a+1,n}(\epsilon'), V - F_{a+1} \right\} dG(\epsilon') + \delta V.$$

A productive firm produces $\mu_n \epsilon$ and pays wages $w_{a,n}(\epsilon)$ in the current period. The following period, which is discounted at rate β, the firm receives the expected continuation value of being in a match with a worker one period older, if the worker does not retire. If the worker retires, the firm becomes vacant. We denote by $\widetilde{J}_{a,n}$ the firm's expected continuation value of an existing match of quality μ_n with a worker of age a.[11] This value depends on the probability that the match is exogenously destroyed, δ, and on the expected value of idiosyncratic productivity ϵ. The firm decides, given ϵ, whether to continue with the match, or to dissolve it and pay the firing cost F_a.

If the firm is starting a new employment relationship with a worker, it needs to pay an initial cost c before starting production. Hence,

$$J^0_{a,n}(\epsilon) = \mu_n \epsilon - w^0_{a,n}(\epsilon) - c + \beta \left[\mathbb{I}_{a<T} \widetilde{J}_{a+1,n} + (1 - \mathbb{I}_{a<T}) V \right], \tag{3}$$

$$J^0_a(\epsilon) = \left(\sum_{n=1}^{N} \pi_n \mu_n \right) \epsilon - w^0_a(\epsilon) - c + \beta \sum_{n=1}^{N} \pi_n \left[\mathbb{I}_{a<T} \widetilde{J}_{a+1,n} + (1 - \mathbb{I}_{a<T}) V \right], \tag{4}$$

[11] Note that although $\mathbb{I}_{a<T} = 0$ for $a = T$, technically we still need to define the expected continuation value for a firm that is hiring a retiring worker. We set this value to $\widetilde{J}_{T+1,n} = V$.

where $w_a^0(\epsilon)$ is the initial wage, negotiated based on the expected, not the actual, value of μ.

The Problem of the Worker

A worker looks for a job while in the last year of school or when unemployed. The value of being unemployed for a worker of age a is:[12]

$$\begin{aligned} U_a &= b + \mathbb{I}_{a<T}\beta \Bigg\{ \theta q(\theta)(1-\delta) \Bigg(R_{a+1} \sum_{n=1}^{N} \pi_n \int_{\epsilon_{min}}^{\epsilon_{max}} \max\left\{W_{a+1,n}^0(\epsilon'), U_{a+1}\right\} dG(\epsilon') \\ &\quad + (1-R_{a+1}) \int_{\epsilon_{min}}^{\epsilon_{max}} \max\left\{W_{a+1}^0(\epsilon'), U_{a+1}\right\} dG(\epsilon') \Bigg) + \left[1-\theta q(\theta)(1-\delta)\right] U_{a+1} \Bigg\}. \end{aligned} \tag{5}$$

An unemployed worker receives a flow value b, which captures the value of leisure, home production, and unemployment benefits. The following period, if the worker does not retire, the worker is matched with a firm with probability $\theta q(\theta)$, and if the match is not exogenously destroyed, the worker decides whether to start working for a firm or to remain unemployed. The value of employment for the worker depends on whether the match quality is immediately revealed, and on the value of the idiosyncratic productivity.

We denote the value of employment at a newly formed match for a worker of age a as $W_{a,n}^0(\epsilon)$ and $W_a^0(\epsilon)$, depending on whether the match quality is immediately revealed and of value μ_n, or is not yet known. The expressions for these two values are:

$$W_{a,n}^0(\epsilon) = w_{a,n}^0(\epsilon) + \mathbb{I}_{a<T}\beta \widetilde{W}_{a+1,n}, \tag{6}$$

$$W_a^0(\epsilon) = w_a^0(\epsilon) + \mathbb{I}_{a<T}\beta \sum_{n=1}^{N} \pi_n \widetilde{W}_{a+1,n}, \tag{7}$$

where $w_{a,n}^0(\epsilon)$ and $w_a^0(\epsilon)$ are the wages paid by the firm, and $\widetilde{W}_{a,n}$ is the expected continuation value of an employment relationship of quality μ_n for a worker of age a and who is

[12] If an individual is matched with a firm while in school, he stops searching and waits until the age of 1, at which moment he becomes either employed, or unemployed in the unlucky case of an immediate dissolution of the match. Therefore there is no decision to be made before an individual enters the labor market, hence it is not necessary to write the value of search for a person still in school.

not retiring,[13] and which is defined as

$$\widetilde{W}_{a+1,n} = (1-\delta) \int_{\epsilon_{min}}^{\epsilon_{max}} \max\left\{W_{a+1,n}\left(\epsilon'\right), U_{a+1}\right\} dG\left(\epsilon'\right) + \delta U_{a+1}.$$

The previous expression depends on the value of employment for a worker of age a, at an ongoing match with quality μ_n and idiosyncratic productivity ϵ, which is given by an equation very similar to those for newly formed matches:

$$W_{a,n}(\epsilon) = w_{a,n}(\epsilon) + \mathbb{I}_{a<T}\beta\widetilde{W}_{a+1,n}. \tag{8}$$

3.3 Surpluses, Wages, Thresholds, and Flows

Surplus

When a firm and a worker enter into an employment relationship, the match generates a surplus that is defined as the sum of what both agents gain from the match minus what they lose: $S_{a,n}(\epsilon) \equiv J_{a,n}(\epsilon) + W_{a,n}(\epsilon) - U_a - V + F_a$, $S^0_{a,n}(\epsilon) \equiv J^0_{a,n}(\epsilon) + W^0_{a,n}(\epsilon) - U_a - V$, and $S^0_a(\epsilon) \equiv J^0_a(\epsilon) + W^0_a(\epsilon) - U_a - V$.

Since the idiosyncratic productivity of the match, ϵ, is i.i.d. and drawn every period, the continuation values for the different states of the agents do not depend on the current value of ϵ. Using the previous definitions of the match surplus as well as the value functions from above, it is easy to show that the surplus of the match is a strictly increasing function of ϵ. Given the assumption of wages being set as the Nash solution to a bargaining problem, it is also evident that the model satisfies the reservation productivity property, as is standard in this literature. In other words, there exists a single threshold for ϵ, below which the termination of the match is optimal for both the firm and the worker. Such a threshold corresponds to the value of ϵ that makes the match surplus zero. These thresholds, whose expressions are stated later, are denoted as $\bar{\epsilon}_{a,n}$, $\bar{\epsilon}^0_{a,n}$ and $\bar{\epsilon}^0_a$ for the surpluses $S_{a,n}(\epsilon)$, $S^0_{a,n}(\epsilon)$ and $S^0_a(\epsilon)$, respectively.

[13] As in the problem of the firm, we need to define the expected continuation value for a worker who is retiring for equations (6) and (8) to be well-defined for $a = T$. We set this value to $\widetilde{W}_{T+1,n} = W^0_{T+1}(\epsilon) = W^0_{T+1,n}(\epsilon) = U_{T+1} = 0\ \forall\epsilon$.

Using the previous value functions, we obtain the following expressions for the surpluses:

$$\begin{aligned} S_{a,n}(\epsilon) &= \mu_n \epsilon - b + F_a - \mathbb{I}_{a<T}\beta(1-\delta) F_{a+1} + \mathbb{I}_{a<T}\beta(1-\delta)\int_{\bar{\epsilon}_{a+1,n}}^{\epsilon_{max}} S_{a+1,n}(\epsilon') \, dG(\epsilon') \\ & -\mathbb{I}_{a<T}\beta\theta q(\theta)(1-\delta)\left(R_{a+1}\sum_{n=1}^{N}\pi_n \int_{\bar{\epsilon}^0_{a+1,n}}^{\epsilon_{max}} \left[W^0_{a+1,n}(\epsilon') - U_{a+1}\right] dG(\epsilon') \right. \qquad (9) \\ & \left. + (1-R_{a+1})\int_{\bar{\epsilon}^0_{a+1}}^{\epsilon_{max}} \left[W^0_{a+1}(\epsilon') - U_{a+1}\right] dG(\epsilon')\right), \end{aligned}$$

$$S^0_{a,n}(\epsilon) = S_{a,n}(\epsilon) - c - F_a, \qquad (10)$$

$$S^0_a(\epsilon) = \sum_{n=1}^{N}\pi_n S_{a,n}(\epsilon) - c - F_a. \qquad (11)$$

Note that starting from $a = T$, we can recursively establish up to $a = 1$ that $S_{a,n}(\epsilon)$ is strictly increasing in μ_n. This is natural, since the current match quality does not affect the worker's outside option value.

Wages

Wages are determined according to generalized Nash bargaining between the matched firm and worker. Therefore, wages for an ongoing match solve the following optimal problem:

$$\max_{w_{a,n}(\epsilon)} \left(J_{a,n}(\epsilon) - V + F_a\right)^{1-\eta}\left(W_{a,n}(\epsilon) - U_a\right)^{\eta},$$

where η is the bargaining power of the worker. The wages on newly formed matches, $w^0_{a,n}(\epsilon)$ and $w^0_a(\epsilon)$, solve a similarly defined problem except there are not firing costs. The solution to the previous problem delivers a sharing rule that implies that both firm and worker get a constant share of the surpluses equal to their bargaining power (i.e., $J_{a,n}(\epsilon) - V + F_a = (1-\eta) S_{a,n}(\epsilon)$ and $W_{a,n}(\epsilon) - U_a = \eta S_{a,n}(\epsilon)$). The expression for wages are:

$$\begin{aligned} w_{a,n}(\epsilon) = &\ \eta\left[\mu_n\epsilon + F_a - \mathbb{I}_{a<T}\beta(1-\delta)F_{a+1}\right] \qquad (12) \\ & + (1-\eta)\left[b + \mathbb{I}_{a<T}\beta\theta q(\theta)(1-\delta)\eta\left(R_{a+1}\sum_{n=1}^{N}\pi_n\int_{\bar{\epsilon}^0_{a+1,n}}^{\epsilon_{max}} S^0_{a+1,n}(\epsilon')\,dG(\epsilon')\right.\right. \\ & \left.\left. + (1-R_{a+1})\int_{\bar{\epsilon}^0_{a+1}}^{\epsilon_{max}} S^0_{a+1}(\epsilon')\,dG(\epsilon')\right)\right], \end{aligned}$$

$$w_{a,n}^{0}\left(\epsilon\right) = w_{a,n}\left(\epsilon\right)-\eta\left(c+F_{a}\right), \tag{13}$$

$$w_{a}^{0}\left(\epsilon\right) = \sum_{n=1}^{N}\pi_{n}w_{a,n}\left(\epsilon\right)-\eta\left(c+F_{a}\right). \tag{14}$$

Threshold

The separation thresholds are defined as the level of idiosyncratic productivity that makes the surplus of the match equal to zero. For existing matches, the threshold is:

$$\begin{aligned}\bar{\epsilon}_{a,n} = & \frac{1}{\mu_{n}}\Bigg\{b-F_{a}+\mathbb{I}_{a<T}\beta\left(1-\delta\right)F_{a+1}-\mathbb{I}_{a<T}\beta\left(1-\delta\right)\int_{\bar{\epsilon}_{a+1,n}}^{\epsilon_{max}}S_{a+1,n}\left(\epsilon'\right)dG\left(\epsilon'\right)\\ & +\mathbb{I}_{a<T}\beta\theta q\left(\theta\right)\left(1-\delta\right)\eta\left(R_{a+1}\sum_{n=1}^{N}\pi_{n}\int_{\bar{\epsilon}_{a+1,n}^{0}}^{\epsilon_{max}}S_{a+1,n}^{0}\left(\epsilon'\right)dG\left(\epsilon'\right)\right.\\ & \left.+\left(1-R_{a+1}\right)\int_{\bar{\epsilon}_{a+1}^{0}}^{\epsilon_{max}}S_{a+1}^{0}\left(\epsilon'\right)dG\left(\epsilon'\right)\right)\Bigg\}.\end{aligned} \tag{15}$$

Since $S_{a+1,n}$ is increasing in the match quality μ_n as mentioned earlier, these expressions imply that the destruction thresholds are decreasing in μ_n. Therefore, workers who are in a better quality match are less likely to leave the match.

For newly created matches, the threshold is given by

$$\bar{\epsilon}_{a,n}^{0} = \bar{\epsilon}_{a,n}+\frac{1}{\mu_{n}}\left(c+F_{a}\right) \tag{16}$$

when the match quality is immediately revealed, and

$$\bar{\epsilon}_{a}^{0}=\frac{1}{\sum_{n=1}^{N}\pi_{n}\mu_{n}}\left(\sum_{n=1}^{N}\pi_{n}\mu_{n}\bar{\epsilon}_{a,n}+c+F_{a}\right), \tag{17}$$

when the match quality is not revealed.[14]

Flows

We normalize the population for every age group to unity, and assume that workers start searching for jobs in the last period of school. Since the model is quarterly, this implies that the number of searchers at age $a=-3$ is $s_{-3}=1$. For the following three quarters, students keep searching for jobs if they have not matched with a firm, which implies

[14] The values computed using equations (15)–(17) may lie outside the domain of the idiosyncratic productivity, $[\epsilon_{\min},\epsilon_{\max}]$. In such cases, we restrict the threshold value to equal the adequate boundary.

that $s_a = [1 - \theta q(\theta)] s_{a-1}$ for $a \in \{-2, ..., 0\}$. In turn, this means that the number of unemployed workers at age zero is $u_0 = s_0$. Using the finding and separation rates derived from the above conditions, we can express the stock of workers in each of the possible states for $a \in \{1, ..., T-1\}$ as:

$$
\begin{aligned}
1 &= u_a + e_a, \qquad (18)\\
u_a &= \left[1 - \theta q(\theta)(1-\delta)\left\{R_a \sum_{n=1}^{N} \pi_n \left(1 - G\left(\bar{\epsilon}^0_{a,n}\right)\right) + (1 - R_a)\left(1 - G\left(\bar{\epsilon}^0_a\right)\right)\right\}\right] u_{a-1}\\
&\quad + \sum_{n=1}^{N} \left[\delta + (1-\delta) G\left(\bar{\epsilon}_{a,n}\right)\right] e_{a-1,n}, \qquad (19)\\
e_a &= e_{a-1} - \sum_{n=1}^{N} \left(\delta + (1-\delta) G\left(\bar{\epsilon}_{a,n}\right)\right) e_{a-1,n}\\
&\quad + \theta q(\theta)(1-\delta)\left\{R_a \sum_{n=1}^{N} \pi_n \left(1 - G\left(\bar{\epsilon}^0_{a,n}\right)\right) + (1 - R_a)\left(1 - G\left(\bar{\epsilon}^0_a\right)\right)\right\} u_{a-1}, \qquad (20)\\
e_{a,n} &= \pi_n \theta q(\theta)(1-\delta)\left\{R_a \left(1 - G\left(\bar{\epsilon}^0_{a,n}\right)\right) + (1 - R_a)\left(1 - G\left(\bar{\epsilon}^0_a\right)\right)\right\} u_{a-1}\\
&\quad + e_{a-1,n}(1-\delta)\left(1 - G\left(\bar{\epsilon}_{a,n}\right)\right), \qquad (21)\\
e_a &= \sum_{n=1}^{N} e_{a,n}, \qquad (22)
\end{aligned}
$$

where u_a and e_a are the number of unemployed and employed workers of age a, and $e_{a,n}$ is the number of employed workers of age a in a match with quality μ_n.

The aggregate unemployment and employment rates are:

$$
\begin{aligned}
u &= \frac{1}{T} \sum_{a=1}^{T} u_a, \qquad (23)\\
e &= 1 - u. \qquad (24)
\end{aligned}
$$

Equilibrium

We assume that there is free entry of firms into the labor market in equilibrium. This implies that the equilibrium value of posting a vacancy is zero, $V = 0$, which in turns implies the following condition for vacancy posting:

$$k = \beta q(\theta)\left\{\sum_{a=-3}^{-1}\gamma_a\beta^{-a}\left[R_1\sum_{n=1}^{N}\pi_n\widetilde{J}_{1,n}^0+(1-R_1)\widetilde{J}_1^0\right]\right.$$
$$\left.\sum_{a=0}^{T-1}\gamma_a\left[R_{a+1}\sum_{n=1}^{N}\pi_n\widetilde{J}_{a+1,n}^0+(1-R_{a+1})\widetilde{J}_{a+1}^0\right]\right\}. \quad (25)$$

A stationary market equilibrium in this economy is a set of $\left\{S_{a,n}(\epsilon), S_{a,n}^0(\epsilon), S_a^0(\epsilon), \bar{\epsilon}_{a,n}, \bar{\epsilon}_{a,n}^0, \bar{\epsilon}_a^0, \theta, e_{a,n}, e_a, u_a\right\}$ for $a \in \{1, 2, \ldots, T\}$ and $n \in \{1, 2, \ldots, N\}$ such that (i) the surpluses are consistent with the agents' problems and satisfy equations (9) to (11); (ii) separation decisions are individually efficient and satisfy equations (15) to (17); (iii) expected profit from posting a vacancy is zero, and satisfies equation (25); (iv) the probabilities of finding a worker/firm are consistent with the matching function; and (v) the implied employment and unemployment are consistent with the above conditions and satisfy the flow equations (18) to (24).

Existence and uniqueness of the equilibrium are confirmed by solving the equilibrium equations backwards, starting from $a = T$.

4 Results

In order to examine the quantitative implications of the model, we calibrate it to match key facts of the Japanese labor market for the period 1983-2008. We then compare the life-cycle implications of the model with the patterns observed in actual data.

4.1 Parameterization

We set a model period to one quarter, and set the discount factor β to 0.99. We assume working life to be 50 years, corresponding to age 15–64. This implies $T = 200$, and workers reach the mandatory retirement age of 60 at $a = 181$.

We assume that the matching function is Cobb-Douglas, $m(u, v) = \kappa u^{\alpha} v^{1-\alpha}$. Both the worker's bargaining power η and the unemployment elasticity of matches α are set to 0.5, which are conventional choices in the literature. There is no consensus in the literature on the distribution of ϵ, so for simplicity, we assume it follows a uniform distribution, $\epsilon \sim U[0, 1]$.

The remaining parameters are determined such that the steady state of the model matches specific long-run labor market facts for Japan. Following Shimer (2005), we normalize θ to equal 1 in the steady state, and choose the vacancy cost k such that the zero profit condition for posting a vacancy, eq (25), holds for $\theta = 1$. We set the training cost c to 1.48 times quarterly average output, following Merz and Yashiv (2007)'s estimate of marginal hiring cost in the U.S.[15] We choose the value of b to be 71 percent of average wage, following Hall and Milgrom (2008). We select the scaling parameter of the matching function κ to match the unemployment rate of the youngest age group, 8.78 percent, and the exogenous separation rate δ to match the separation rate of the same group, 3.91 percent.

While it is widely accepted that the dismissal of workers is difficult in Japan, quantitative evaluation of firing costs is a complex task that has not been performed in the literature. There is a further complication due to a common practice, explained in Section 2.2, by which workers who have reached retirement age are often reemployed on a fixed-term basis, hence with less job security. Here, we assume that $F_a = f_y$ for workers of age $a < T - 20$, and $F_a = f_o$ for those of age $a \geq T - 20$; we then choose f_y such that firing costs equal two quarters of the average wage for workers before the age of mandatory retirement, and set f_o to $f_y/6$, corresponding to firing costs of average monthly wage for workers in the last five years in the labor market.[16]

The parameters related to match quality are chosen as follows. We assume that the probability of immediate revelation of match quality R_a declines linearly with age a: $R_a = r_a (a-1)$ for $a \in \{1, 2, \ldots, T\}$. The coefficient r_a is calibrated to match the average of job separation rates for the ten five-year age groups, 1.58 percent. The idea behind this formulation of R_a is that, prior to entering an employment relationship, it is difficult to assess the match quality of young workers who have little employment history. As workers accumulate labor market experience, such assessment should become easier. We assume that μ_n lies on $N = 20$ grid points,[17] equispaced on the interval $[\mu_1, \mu_N]$, with equal probability. We normalize μ_1 to 1, and calibrate μ_N to match the average unemployment rate for all age groups for the period of study, 4.15 percent.

Table 1 summarizes the resulting parameters.

[15] Hahn (2009) uses the same calibration target for this variable. This value is larger than 32 percent of the quarterly average wage, the target used in Esteban-Pretel and Fujimoto (2010) for the U.S. economy, based on the discussion in Mortensen (1994). In this paper, we adopt this larger target value, in order to reflect the widely accepted view that Japanese firms exert large efforts in training their employees.

[16] We consider this to be more appropriate than assuming zero firing costs for those workers over the mandatory retirement age, since workers on fixed-term contracts, while enjoying far less job security than regular workers, cannot be dismissed before the end of the contracts, except in unavoidable circumstances (Labor Contract Law, Article 17(1)).

[17] Our numerical results are not very sensitive to the value of N as long as it is sufficiently large.

4.2 Simulation Results

We now compare the unemployment, separation, and job-finding rates predicted by the model with those in the Japanese data. Since the model is quarterly, we take the averages for 20-period groups for the model results, in order to have them correspond with their empirical counterparts shown in Section 2, which are averages for five-year age groups. To facilitate the explanation of key mechanisms behind the model predictions, we present the results in the following order: job-finding rate, job separation rate, and unemployment rate.

Figure 2c shows the job-finding rate in the model and in the data. While the parameterization of the model targets none of the job-finding rates in the data, the model successfully generates a decline in this rate as workers age, and matches the data very well. Combination of a finite horizon and uncertainty regarding the quality of matches is key to delivering a decline in this rate.

Since workers retire in a finite number of periods, the expected value of hiring a worker declines with worker age, all other things being equal. This is the horizon effect. All workers have equal probabilities of being matched with firms, since we assume a single labor market. However, the shorter horizons of older workers make them less valuable to the firm, and accordingly, firms are more selective when hiring older workers. The probability that a created match is immediately dissolved thus increases with worker age, which reduces the finding rate for older workers.

We assume that the probability of immediate match quality revelation is higher for older workers. The matches that are immediately revealed to be of poor quality are promptly dissolved and do not become employment relationships, lowering the finding rate for older workers. In contrast, the quality of matches with young workers is less likely to be immediately revealed, and this provides incentives for both firms and workers to postpone match dissolution and proceed into the employment relationship.

While the horizon effect is qualitatively important, it is the existence of match quality, together with the increasing probability, as worker age increases, of its immediate revelation, that quantitatively dominates the simulation results.

Figure 2b plots the job separation rates in the model and in actual data. The calibration targets the separation rate of the youngest age group, and it also separately targets the average of separation rates for ten age groups. The separation rates for age groups other than the youngest one are determined endogenously in the model. We observe that the model manages to capture the U-shaped nature of the age-separation profile, and fits the data fairly well.

In isolation, the horizon effect yields separation rates that increase with age, via the same mechanism as for the declining job-finding rates. This effect, combined with the lower firing cost for the oldest age group, generates in the model the rise in separation rate from the second-oldest to the oldest age group observed in the actual data. Furthermore, the inclusion of random match quality, combined with horizon effects, causes the model to reproduce the declining job separation rates for other age groups in the data, due to the following mechanisms.

The match quality is unknown for many newly-formed matches, but is revealed after a period of employment, at which point a large proportion of low-quality matches are terminated. Young workers entering the labor market without a job are likely to be in their first period of employment, the match quality of which may be unknown. Furthermore, compared to matches with older workers, a higher proportion of newly-formed matches with younger workers are of unknown match quality. Finally, the horizon effect makes the opportunity cost of staying in a low-quality match higher for young workers than for older ones, causing matches with younger workers to be more likely to be terminated. The combination of these effects produces separation rates in the model that decline with age, as observed in the data.

Compared to its empirical counterpart, however, the predicted separation rate declines more from the youngest to the second-youngest age group. This is a product of the extreme feature of the model that match quality is revealed after one period of tenure, whereas in reality, match quality is likely to be perceived over time. The model prediction in this dimension might be improved through the incorporation of some learning process, as for example in Pries and Rogerson (2005).[18]

Finally, Figure 2a plots the unemployment rate in the model and in the Japanese data. As discussed above, our calibration targets the average unemployment rate and the unemployment rate for the youngest age group, but does not directly target the unemployment rates for the other age groups. Figure 2a shows that the model predicts the life-cycle pattern of unemployment rates quite well. The model delivers the sharp decline in unemployment for workers in the early years of their working lives, the much slower decline after the age of 30, and the rise in unemployment for ages 60 and above. Such model predictions for the unemployment rate can be understood as the combined effects of the separation and job-finding rates. Workers enter the labor market without a job, and despite high find-

[18] In Pries and Rogerson (2005), endogenous separation occurs only in the period in which match quality is revealed; all separations beyond that period are exogenous. Introducing learning into our framework, in which endogenous separation occurs throughout the match due to fluctuations in its idiosyncratic productivity, is a challenging task since the presence of two different sources of uncertainty complicates the learning mechanism.

ing rates, are more likely to quickly separate from firms and become unemployed. These three factors combine to produce higher unemployment rates for young workers, which decline as workers age, find higher-quality matches, and separate less often. Furthermore, a decline in the job-finding rate, combined with a rise in separation rate, generates a high unemployment rate for the oldest age group.

Importantly, the predicted levels of the all three variables of interest are close to those in the actual data for the youngest age group. While we are calibrating the model to obtain such a fit for the separation and unemployment rates, the model endogenously yields such prediction for the job-finding rate, and our assumption that workers start searching for a job before entry into the labor market turns out to be crucial in achieving this result. In fact, without this assumption, the predicted job-finding rate for the youngest age group is far too high compared to the data.

5 Labor Market Changes Due to Productivity Decline

During the 1990s, Japan suffered one of the most severe recessions of the post-war world economy; many hypotheses have attempted to explain this deep and prolonged downturn. One theory that has drawn considerable attention is developed in Hayashi and Prescott (2002), in which a severe drop in productivity is posited as the cause of the recession. While the model in Hayashi and Prescott (2002) does not include unemployment, Esteban-Pretel, Nakajima, and Tanaka (2010) recently showed that the decline in productivity experienced in Japan during the 1990s can account for the increase in aggregate unemployment observed in that period. Similarly, Ariga and Okazawa (2011) also build a search model of the labor markets and study the effects of the productivity changes in the 1990s on aggregate unemployment. Neither of the latter two papers use a life-cycle model, however.

Both Hayashi and Prescott (2002) and Esteban-Pretel, Nakajima, and Tanaka (2010) assume that the economy was in an initial steady state during the 1980s, and that the productivity drop moved it to a different steady state. We follow this approach and test if our model, which matches the long-run life-cycle features of the Japanese labor market well, also reproduces the changes in three variables of interest from the 1980s to the 2000s.

The exercise we perform consists of recalibrating the model to the 1980s averages (particularly 1983Q1–1990Q4), where the overall unemployment and the separation rates were low (2.5 and 1.02 percent on average, respectively) and the finding rate was high (36.7 percent). Then, keeping all parameters at the 1980s level, we reduce productivity by 5 and 10 percent, close to estimates of the average drop in detrended productivity between the

1990s and the 2000s,[19] and study if the model is able to capture the rises in unemployment and separation rates and the drop in the finding rate observed in Japan during the 2000s.[20] More precisely, we assume that the flow output of a match is now $p\mu\epsilon$, where p is aggregate productivity, taking the value of $p = 1$ in the initial steady state and $p = 0.95$ and $p = 0.9$ in the final ones.

Figure 3c plots the job-finding rate in the model for these three values of p, together with the data for 1983Q1–1990Q4 and 2002Q1–2008Q4. We observe that reducing p in the model generates a decline in the job-finding rate, as is the case in the data for most age groups from the 1980s to the 2000s. The model decline in the finding rate is due to the drop in profits from a match following the fall in productivity, which reduces the incentive for firms to post vacancies. Numerically, the result for $p = 0.95$, or a 5 percent drop in productivity, better captures the empirical change in the job-finding rate than that caused by the larger productivity drop.

The results for the separation rate are shown in Figure 3b. In this case, we observe that varying the value of p has almost no impact on the separation rate. This is likely the result of the presence of firing cost in the model, and the decline in the workers' outside option values resulting from the lower job-finding rate.[21]

Finally, Figure 3a displays the results for the unemployment rate. We see that the model generates the rise in unemployment rate when p drops, as observed in the data. It is the larger drop in productivity, 10 percent, that better captures the actual fall in unemployment, unlike for the finding rate. This is because, following the simulated productivity drop, unemployment in the model solely increases due to the decline in the job-finding rate, whereas in the data both the job-finding and separation rates cause the rise in unemployment. A larger drop in the finding rate is hence required to match the actual unemployment increase.

6 Conclusions

The unemployment, job-finding, and separation rates are far from homogeneous for workers of different ages. In Japan, the unemployment and separation rates display a U-shape, with young workers having the highest rates, followed by workers close to retirement. The

[19]The measure of productivity used is output per worker. Productivity dropped by around 9 percent from the 1983Q1-1990Q4 period to the 2002Q1-2008Q4 period if detrended at 2.4 percent per year, the average growth rate between 1983Q1 to 1990Q4.

[20]The precise period used to proxy for the final steady state is from 2002Q1, when unemployment peaked, to 2008Q4, which is the end of our sample.

[21]Prat (2007) finds a similar channel, referred to as the *outside option effect*, through which a fall in growth rate, rather than the level of productivity, reduces separations and thus the unemployment rate.

finding rate declines with age, indicating the difficulties faced by older workers who lose their jobs towards the end of their careers.

We develop a life-cycle search and matching model of the labor market that features random match quality and incorporates certain elements capturing the characteristics of the Japanese labor market. We find that our model, calibrated to the Japanese economy, reproduces the life-cycle properties observed in the data. Both the horizon effect and the uncertainty of the match quality are crucial in delivering the fit between the model and the data. We also show that a drop in productivity, similar in size to that observed in Japan since the 1990s, produces changes in the unemployment and finding rates in the model of comparable magnitude to those in the Japanese economy.

The framework developed in this paper, which is stylized but incorporates many realistic components, can be used to study various labor market issues, such as policies related to youth unemployment, mandatory retirement age, and firing restrictions.

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Table 1: Calibrated parameter values

Exogenous Parameters

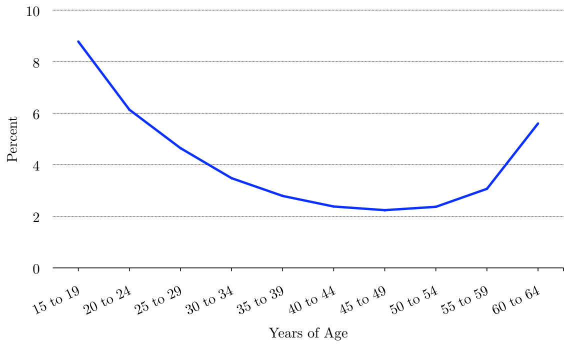
Discount factor	β	0.99
Worker's bargaining power	η	0.5
Unemployment elasticity of matches	α	0.5
Minimum match quality (normalization)	μ_1	1
Distribution of ϵ	G	$U[0,1]$

Endogenous Parameters

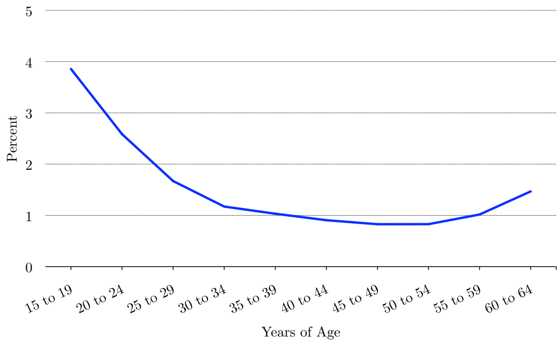
Scale of matching function	κ	0.44
Cost of posting a vacancy	k	0.1512
Initial training cost	c	0.981
Unemployment benefit	b	0.4423
Exogenous separation rate	δ	0.0088
Firing costs for workers before the mandatory retirement age	f_y	1.246
Firing costs for workers after the mandatory retirement age	f_o	0.2077
Coefficient in the probability of match quality revelation	r_a	0.0033
Maximum match quality	μ_N	1.463

Figure 1: Japan's Unemployment, Separation, and Finding Rates by Age

(a) Unemployment Rate



(b) Quarterly Separation Rate



(c) Quarterly Finding Rate

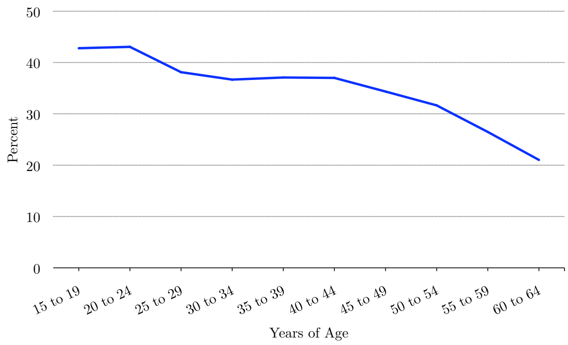
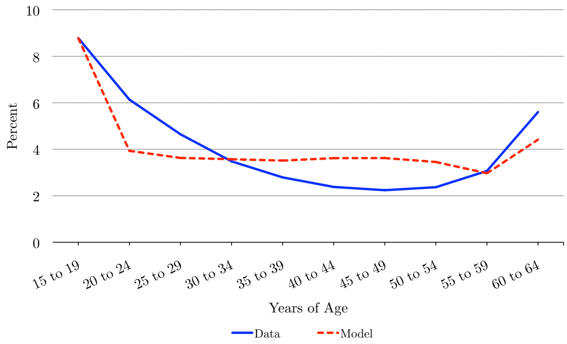
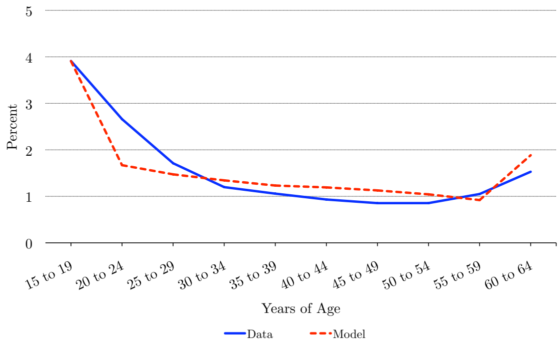


Figure 2: Japan's Unemployment, Separation, and Finding Rates in the Data and the Model

(a) Unemployment Rate



(b) Quarterly Separation Rate



(c) Quarterly Finding Rate

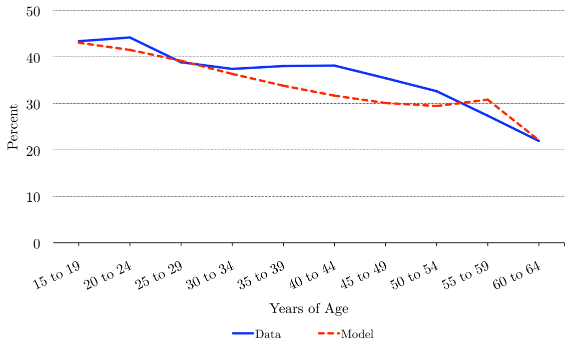
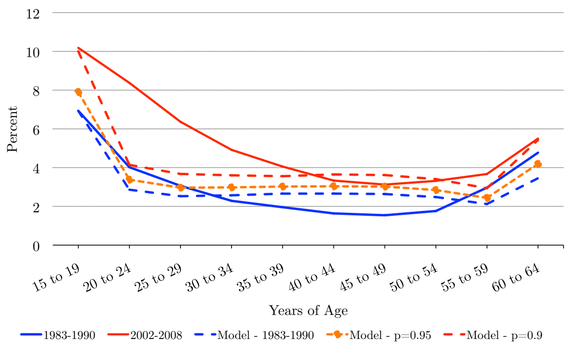
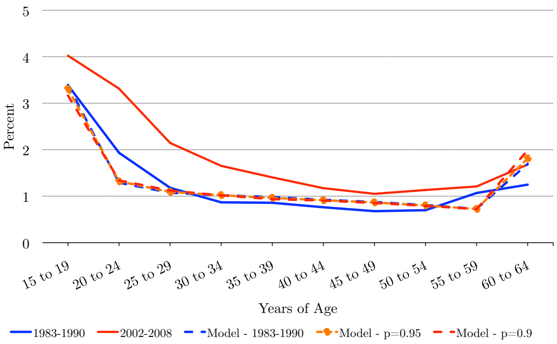


Figure 3: Changes in Productivity

(a) Unemployment Rate



(b) Quarterly Separation Rate



(c) Quarterly Finding Rate