APPENDICES: An application of business cycle accounting with misspecified wedges

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A Model

In this section, we provide descriptions of our prototype model for BCA and the two detailed economies for our experiment.

A.1 Prototype economy

The prototype economy for BCA is almost the same as that employed by CKM.[1] The log-linearized system is as follows.

The intratemporal optimization condition[2] is

$$c_t + \frac{\ell}{1-\ell}\ell_t = y_t - \ell_t - \tau_{\ell,t}, \tag{1}$$

where c_t denotes consumption; ℓ_t, labor supply; y_t, output; and $1-\tau_{\ell,t}$, the investment wedge.

The Euler equation is

$$\begin{aligned}&(1+\tau_x)(\tau_{x,t}-c_t)\\ &\quad = \beta\left\{\alpha\frac{y}{k}\left(E_t y_{t+1}-k_t\right)+(1+\tau_x)(1-\delta)E_t\tau_{x+1}-\left[(1+\tau_x)(1-\delta)+\alpha\frac{y}{k}\right]E_t c_{t+1}\right\},\end{aligned} \tag{2}$$

where k_t denotes the capital stock at the end of period t; β, the discount factor of a household; δ, the depreciation rate of capital; α, the share of capital in production; and $\frac{1}{1+\tau_{x,t}}$, the investment wedge.

The aggregate production function is

$$y_t = \alpha k_{t-1} + (1-\alpha)\ell_t + a_t, \tag{3}$$

where a_t denotes the efficiency wedge and α denotes the cost share of capital stock in production.

[1]The only difference is that we consider a stationary economy.

[2]We employ the period utility function $U = \log(C_t) + \gamma\log(1-L_t)$ following CKM.

The evolution of capital stock is

$$k_t = (1 - \delta)k_{t-1} + \delta i_t, \tag{4}$$

where δ denotes the depreciation rate of capital.

The resource constraint is

$$\frac{c}{y}c_t + \frac{i}{y}i_t + g_t = y_t, \tag{5}$$

where g_t denotes the government wedge.

In many applications of BCA, the wedges are assumed to be exogenous and evolve according to the VAR(1) process:

$$\boldsymbol{s}_{t+1} = \boldsymbol{P}\boldsymbol{s}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{6}$$

where $\boldsymbol{P}$ denotes a constant matrix; $\boldsymbol{s}_t \equiv [a_t, \tau_{\ell,t}, \tau_{x,t}, g_t]'$, a vector of wedges; and $\boldsymbol{\varepsilon}_{t+1}$, a vector of i.i.d. shocks to wedges with mean zero.

A.2 Detailed economy: Medium-scale DSGE economy

Baseline economy: We employ a medium-scale DSGE model as a laboratory for the assessment of BCA. Here, we provide a brief description of our detailed economy.

Our economy is based on that employed by Smets and Wouters (2007). The only difference between our detailed model and the model of Smets and Wouters (2007) is that there are no investment-specific technology shocks in our economy. This is to guarantee that capital stocks in the two economies are the same.[3]

Following Smets and Wouters (2007), we introduce the linearized version of their model here.

[3]We also apply BCA to our economy with investment-specific technology shocks in the paper.

The resource constraint is

$$y_t = \frac{c}{y}c_t + \frac{i}{y}i_t + \frac{r^k k}{y}u_t + \varepsilon_t^g, \tag{7}$$

where ε_t^g is the government expenditure shock.

The consumption Euler equation is

$$\begin{aligned} c_t =& \frac{\lambda}{1+\lambda}c_{t-1} + \frac{1}{1+\lambda}E_t c_{t+1} \\ &+ \frac{(\sigma_c - 1)\left(\frac{w^h \ell}{c}\right)}{\sigma_c(1+\lambda)}(\ell_t - E_t \ell_{t+1}) - \frac{1-\lambda}{\sigma_c(1+\lambda)}(r_t - E_t \pi_{t+1} + \varepsilon_t^b), \end{aligned} \tag{8}$$

where ε_t^b denotes the risk premium shock; λ, the parameter on the external habit; and σ_c, the inverse of the intertemporal elasticity of substitution.

The investment Euler equation is

$$i_t = \frac{1}{1+\beta}i_{t-1} + \frac{\beta}{1+\beta}E_t i_{t+1} + \frac{1}{(1+\beta)\varphi}q_t, \tag{9}$$

where β denotes the discount factor of a household and φ denotes the steady-state elasticity of the capital adjustment cost function.

The capital Euler equation is

$$q_t = \beta(1-\delta)E_t q_{t+1} + \Big[1 - \beta(1-\delta)\Big]E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \varepsilon_t^b), \tag{10}$$

where q_t denotes Tobin's q and δ denotes the depreciation rate of capital.

The aggregate production function is

$$y_t = \phi_p \Big[\alpha k_t^s + (1-\alpha)\ell_t + \varepsilon_t^a\Big], \tag{11}$$

where k_t^s denotes capital service; ε_t^a, total factor productivity; α, the share of capital in production; and ϕ_p, one plus the share of fixed costs in production, reflecting the presence of fixed costs in production, $\phi_p = 1 + \Phi/y$.

The capital service, k_t^s, is

$$k_t^s = k_{t-1} + u_t, \tag{12}$$

where k_{t-1} denotes the capital stock at the end of period $t-1$.

The utilization rate, u_t, is determined by

$$u_t = \frac{1-\psi}{\psi} r_t^k, \tag{13}$$

where ψ denotes a positive function of the elasticity of the capital utilization adjustment cost function and is normalized to be between zero and one.

The evolution of capital stock, k_t, is

$$k_t = (1-\delta)k_{t-1} + \delta i_t. \tag{14}$$

The definition of the price markup, μ_t^p, is

$$\mu_t^p = \alpha \left[k_t^s - \ell_t \right] + \varepsilon_t^a - w_t. \tag{15}$$

The New-Keynesian Phillips curve with partial indexation is

$$\pi_t = \frac{\iota_p}{1+\beta\iota_p}\pi_{t-1} + \frac{\beta}{1+\beta\iota_p}E_t\pi_{t+1} - \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\xi_p[(\phi_p-1)\varepsilon_p+1]}\mu_t^p + \varepsilon_t^p, \tag{16}$$

where ι_p denotes the degree of indexation to past inflation; ξ_p, the degree of price stickiness; ε_p, the curvature of the Kimball goods market aggregator; and ε_t^p, the price markup shock.

The rental rate of capital is

$$r_t^k = -(k_t^s - \ell_t) + w_t. \tag{17}$$

The definition of the wage markup, μ_t^w, is

$$\mu_t^w = w_t - \left[\sigma_\ell \ell_t + \frac{1}{1-\lambda}(c_t - \lambda c_{t-1})\right], \tag{18}$$

where σ_ℓ denotes the elasticity of labor supply with respect to the real wage.

The wage curve with partial indexation is

$$\begin{aligned} w_t = {} & \frac{1}{1+\beta} w_{t-1} + \frac{\beta}{1+\beta}(E_t w_{t+1} + E_t \pi_{t+1}) - \frac{1+\beta\iota_w}{1+\beta}\pi_t \\ & + \frac{\iota_w}{1+\beta}\pi_{t-1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\xi_w[(\phi_w - 1)\varepsilon_w + 1]}\mu_t^w + \varepsilon_t^w, \end{aligned} \tag{19}$$

where ι_w denotes the degree of wage indexation; ξ_w, the degree of wage stickiness; ε_w, the curvature of the Kimball labor market aggregator; and ε_t^w, the wage markup shock.

The monetary policy reaction function is

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t + r_y(y_t - y_t^*)] + r_{\Delta y}[(y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)] + \varepsilon_t^r, \tag{20}$$

where y_t^* denotes the natural output defined in the flexible price-wage economy.

There are six exogenous shocks in this economy. These six driving forces are assumed to follow the following processes:

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a, \tag{21}$$

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b, \tag{22}$$

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a, \tag{23}$$

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p, \tag{24}$$

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w, \tag{25}$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r, \tag{26}$$

where η_t^g, η_t^b, η_t^a, η_t^p, η_t^w, and η_t^r are i.i.d. shocks with mean zero.

Adding investment-specific technology shocks: With investment-specific technology shocks, our medium-scale DSGE economy changes as follows. The investment Euler equation (9) becomes

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t i_{t+1} + \frac{1}{(1+\beta)\varphi} q_t + \varepsilon_t^i, \tag{27}$$

where ε_t^i denotes the investment-specific technology shock. The evolution of the capital stock (14) becomes

$$k_t = (1-\delta)k_{t-1} + \delta i_t + \delta \left[\left(1 + \beta \right) \varphi \right] \varepsilon_t^i. \tag{28}$$

The investment-specific technology shock evolves according to the following process:

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i, \tag{29}$$

where η_t^i is an i.i.d. shock with mean zero. Note that the capital stock in the detailed economy is no longer the same as that in the prototype economy since the evolution of capital stock (28) is different from (14).

B Equivalence results

Here, we provide a brief description of the equivalence results in BCA.

The definition of the equivalence is as follows.

Definition 1. *A detailed model is equivalent to (covered by) a prototype model if the prototype model can achieve all realized sequences of consumption, investment, labor, output, and capital stock generated in the detailed model.*

CKM give the so-called equivalence results, the prototype model covers a large class of frictional detailed models. However, they do not specify the types of stochastic process of the wedges that are necessary for the equivalence; note that the VAR(1) specification is often employed when BCA is applied to the actual data.

The vector of wedges, $\boldsymbol{s}_t$, associated with the detailed model should be described as

$$\boldsymbol{s}_t = \boldsymbol{\Phi}\boldsymbol{x}_t, \tag{30}$$

where $\boldsymbol{\Phi}$ is a constant matrix and $\boldsymbol{x}_t$ is a vector of endogenous and exogenous state variables in the detailed model. The state vector $\boldsymbol{x}_t$ evolves according to

$$\boldsymbol{x}_{t+1} = \boldsymbol{\Psi}\boldsymbol{x}_t + \boldsymbol{\nu}_{t+1}, \tag{31}$$

where $\boldsymbol{\Phi}$ is a constant matrix and $\boldsymbol{\nu}_{t+1}$ is a vector of structural i.i.d. shocks with mean zero.

Using the above structure, Bäurle and Burren (2007) and Nutahara and Inaba (2008) investigate the necessary and sufficient condition for the equivalence in the case where the wedges of the prototype model evolve according to the conventional VAR(1) process. They find that in many DSGE economies, the equivalence results do not hold in this case.[4] The intuition of Nutahara and Inaba (2008) yields that the equivalence results do not hold if the number of independent endogenous and exogenous state variables in the detailed model is greater than the number of wedges in the prototype model.

The equivalence result does not hold between our medium-scale DSGE model and the prototype model. The number of independent state variables in our model is 12: six endogenous states (consumption c_{t-1}, capital k_{t-1}, investment i_{t-1}, wage w_{t-1}, inflation π_{t-1}, nominal interest rate r_t) and six exogenous shocks (technology shocks ε_t^a, risk premium shock ε_t^b, government purchase shock ε_t^g, price-markup shock ε_t^p, wage-markup shock ε_t^w, and investment-specific technology shock ε_t^i).[5]

[4]See Theorem 1 of Nutahara and Inaba (2008) for the necessary and sufficient condition for the equivalence.

[5]We eliminate the monetary policy shock, ε_t^r, since the current nominal interest rate, r_t, contains the information on it.

C Procedure of BCA

C.1 Parameter values of the detailed economy

The parameter values of the detailed economy are given in Table 1. Most parameters are the same as those estimated by Smets and Wouters (2007). Smets and Wouters (2007) estimate these parameters based on the Bayesian method using the data of the postwar U.S. economy. The differences between Smets and Wouters (2007) and the present paper are that (i) we eliminate the trend and (ii) we set the steady-state level of labor as $\frac{1}{3}$.

[Insert Table 1]

C.2 Definition of the true wedges

In the case of the detailed economy without investment-specific technology shocks, the true wedges are calculated using a system that consist of (i) the equilibrium conditions of the detailed economy (7)–(26) and (ii) the equilibrium conditions of the prototype model (1), (2), (3), and (5).

In the case of the detailed economy with investment-specific technology shocks, the evolution of the capital stock in the prototype model (14) is different from that in the detailed economy (28). As such, in addition to the misspecification of the stochastic process of wedges, there is a misspecification of the evolution of capital stock in BCA. In order to focus on the misspecification of the stochastic process of wedges, we define the true wedges using the capital stock generated in the prototype economy.

Letting $\hat{k}_t$ be the capital stock generated in the prototype model, the true wedges are calculated by the system that consist of (i) the equilibrium conditions of the detailed economy (7)–(8) and (11)–(29) and (ii) the equilibrium conditions of the prototype model

(1), (5), and the conditions given below:

$$
\begin{aligned}
&(1+\tau_x)(\tau_{x,t}-c_t) \\
&\quad = \beta\left\{\alpha\frac{y}{k}\left(E_t y_{t+1}-\hat{k}_t\right)+(1+\tau_x)(1-\delta)E_t\tau_{x+1}-\left[(1+\tau_x)(1-\delta)+\alpha\frac{y}{k}\right]E_t c_{t+1}\right\}, \\
&y_t = \alpha\hat{k}_{t-1}+(1-\alpha)\ell_t+a_t, \\
&\hat{k}_t = (1-\delta)\hat{k}_{t-1}+\delta i_t.
\end{aligned}
$$

C.3 Wedge decomposition

Our method of wedge decomposition is the same as that employed by Chari, Kehoe, and McGrattan (2007a). In wedge decompositions, the counterfactual sequences of wedges are constructed as follows. For example, to investigate the contribution of the efficiency wedge, it is assumed to be the same as the measured efficiency wedge with the other wedges being constant over time. The aggregate decision rules are computed under the specification that all wedges except the efficiency wedge are fixed constants and the only uncertainty agents face is over the realization of the efficiency wedge. This is the "theoretically-consistent methodology" that Chari, Kehoe, and McGrattan (2007b) mention.

In the case of "true," we replace the VAR(1) specification of wedges (6) with the true stochastic process of wedges (30) and (31). This true stochastic process of wedges (30) and (31) can be written as the VARMA(p,q) process, and this should be consistent with the results where we employ the prototype model with the VARMA specification of wedges.

D Robustness: Alternative definition of the true wedges

In this paper, we define the true wedges using the capital stock of the prototype economy in the case with investment-specific technology shocks. This is to focus only on the problem of misspecification of the stochastic process of wedges.

Here, we show what happens if we define the true wedges using the capital stock generated in the detailed economy.

Figure 1 shows the true and measured wedges in our medium-scale DSGE economy with investment-specific technology shocks.

[Insert Figure 1]

With investment-specific technology shocks, the difference between the measured and true investment wedges becomes larger than that in the case without investment-specific technology shocks while the other wedges are measured almost correctly. In the case, there should be a mismeasurement of the efficiency wedge since the two capital stocks are different in the prototype economy and the detailed economy. However, Figure 1 shows that this difference is quantitatively quite small.

Table 2 reports the cyclical behavior of the true and measured investment wedges.

[Insert Table 2]

The correlation between the true and measured investment wedges becomes smaller, and the RMSE indicates that the difference between the two investment wedges is about 1.6 percent. However, the measured investment wedge seems to be close to the true one. At least, the measured investment wedge captures the abstract of the dynamics of the true one.

In this case, there are two sources of mismeasurement of the investment wedge: (i) the difference in capital stocks and (ii) the increase in the number of the state variables. We find that the impact from reason (i) is small.

Figure 2 shows the output decomposition by each wedge

[Insert Figure 2]

For efficiency, labor, and investment wedges, the differences between the output predictions of the true and measured wedges are still small.

Table 3 reports the cyclical behavior of the output predictions by the true and measured investment wedges.

[Insert Table 3]

The RMSE is about 1.5 percent. Finally, we conclude that BCA is empirically useful even in this case.

References

[1] Bäurle, G., Burren, D., 2007. A note on business cycle accounting, University of Bern, Discussion Paper 07–05.

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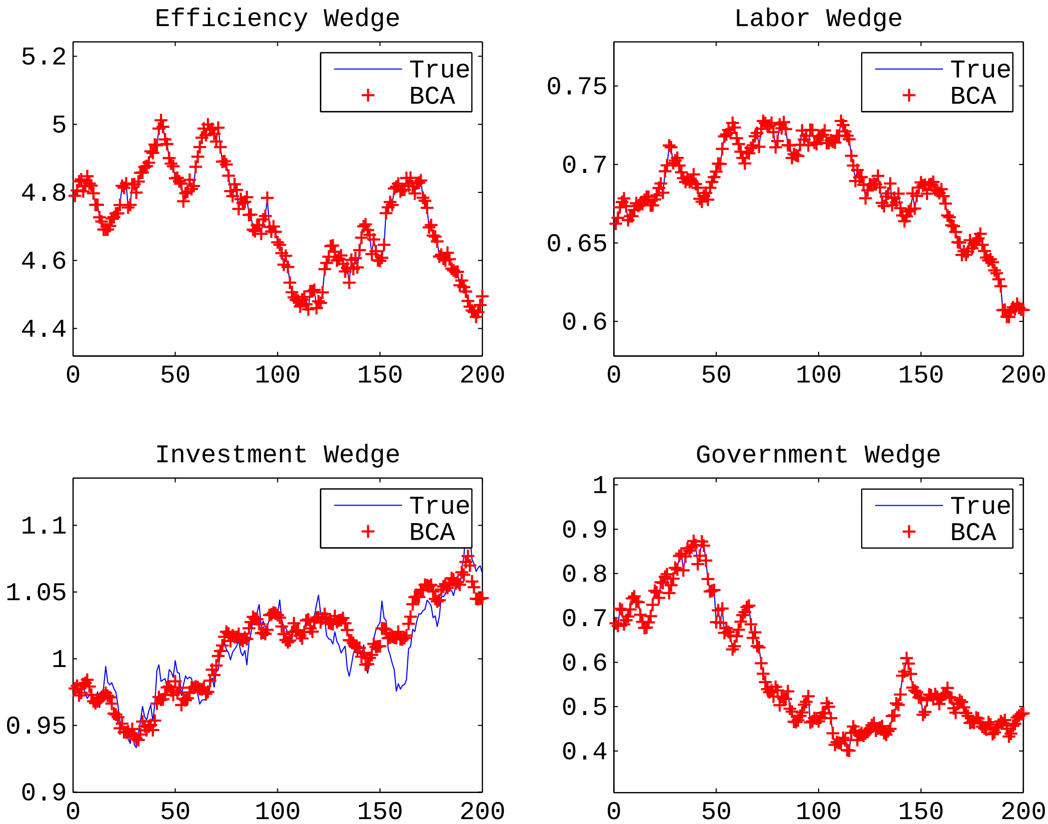
[3] Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2007b. Comparing alternative representations and alternative methodologies in business cycle accounting, Federal Reserve Bank of Minneapolis Staff Report 384.

[4] Nutahara, K., Inaba, M., 2008. On equivalence results in business cycle accounting, RIETI Working Paper 09–E–18.

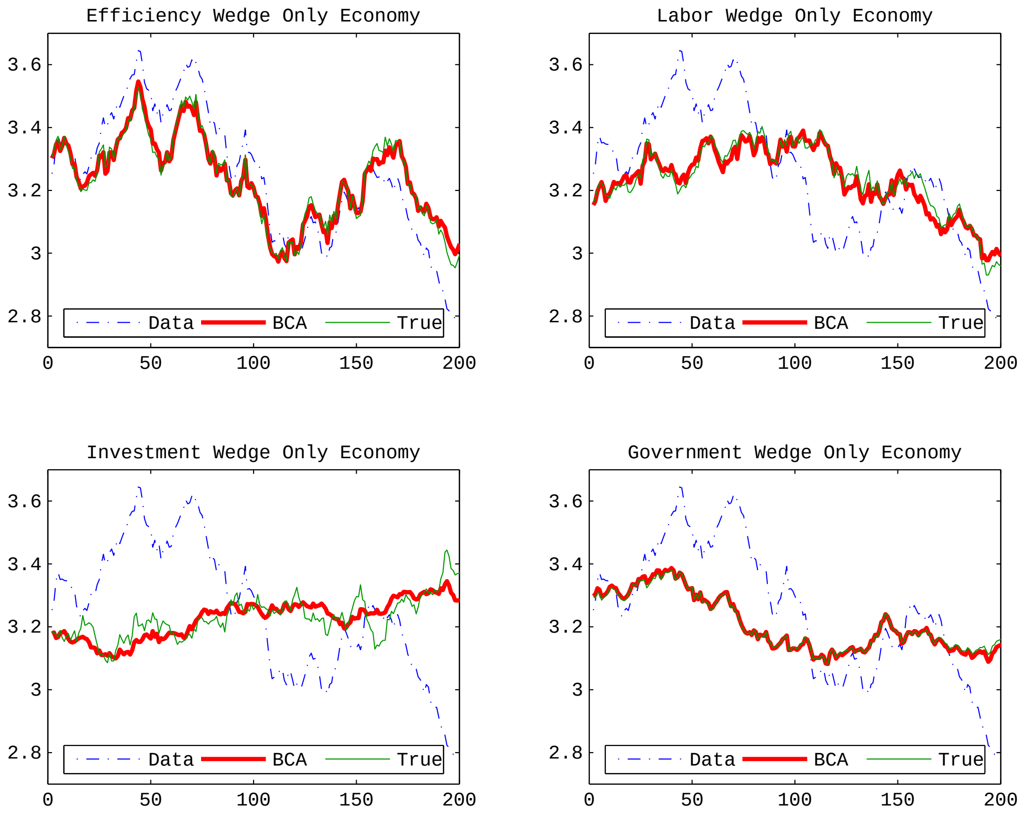
[5] Smets, F., Wouters, R., 2007. Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach. American Economic Review 97, 586–606.

Figure 1: True and measured wedges (3): With investment-specific technology shock (Alternative definition of the true wedges)



Notes: The blue solid lines are the true wedges that are consistent with the Smets-Wouters economy. The red crosses are the measured wedges by BCA. The wedges in the figures are levels (and not log-deviations).

Figure 2: Output decomposition by the true and measured wedge (3): With investment-specific technology shock (Alternative definition of the true wedges)



Notes: The blue dashed-dotted lines are the actual output data. The red bold solid lines are the output predictions by the measured wedges. The green solid lines are the output predictions by the true wedges.

Table 1: Parameter values of the detailed economy

symbol	description	values
σ_c	relative risk aversion	1.39
σ_ℓ	elasticity of labor supply with respect to the real wage	1.92
λ	habit persistence	0.71
ξ_w	wage stickiness	0.73
ξ_p	price stickiness	0.66
ι_w	wage indexation	0.59
ι_p	inflation indexation	0.22
φ	steady-state elasticity of the capital adjustment cost	5.48
ψ	elasticity of the utilization adjustment cost	0.54
Φ	fixed cost in production	1.61
ρ	Taylor rule (past interest rate)	0.81
r_π	Taylor rule (inflation)	2.03
r_y	Taylor rule (output (1))	0.08
$r_{\Delta y}$	Taylor rule (output (2))	0.22
$\bar{\pi}$	steady-state quarterly inflation	0.81
$100(\beta^{-1} - 1)$	discount factor	0.16
$\bar{\ell}$	steady-state labor supply	1/3
α	share of capital	0.19
δ	depreciation rate of capital	0.025
λ_w	steady-state markup in labor market	1.5
ε_p	Kimball aggregators in goods market	10
ε_w	Kimball aggregators in labor market	10
g/y	steady-state ratio of government spending to output	0.18
σ_a	std of technology shock	0.45
σ_b	std of risk-premium shock	0.24
σ_g	std of government shock	0.52
σ_i	std of investment-specific technology shock	0.45
σ_r	std of monetary shock	0.24
σ_p	std of price markup shock	0.14
σ_w	std of wage markup shock	0.24
ρ_a	persistence of technology shock	0.95
ρ_b	persistence of risk-premium shock	0.18
ρ_g	persistence of government shock	0.97
ρ_i	persistence of investment-specific technology shock	0.71
ρ_r	persistence of monetary shock	0.12
ρ_p	persistence of price markup shock	0.90
ρ_w	persistence of wage markup shock	0.97
μ_p	MA parameter of price markup	0.74
μ_p	MA parameter of wage markup	0.88
ρ_{ga}	relationship between technology and government shocks	0.52

Table 2: Cyclical behavior of the true and measured investment wedges (3): With investment-specific technology shock (Alternative definition of the true wedges)

	mean	std	autocorr.	corr. w/ y_t	corr. w/ true	RMSE
true	1.0018	0.0314	0.9671	-0.1192	–	–
VAR(1)	1.0027	0.0231	0.9790	-0.1034	0.8630	0.0164

Notes: Means, standard deviations, autocorrelations, correlations with the current output, correlations with the true investment wedge, and the RMSE are reported. The RMSE is the root mean squared error of the percentage-deviations between the true and measured investment wedges.

Table 3: Cyclical behavior of the predicted output by the true and measured investment wedges (3): With investment-specific technology shock (Alternative definition of the true wedges)

	mean	std	autocorr.	corr. w/ true	RMSE
true	3.2224	0.0649	0.9453	–	–
measured	3.2166	0.0398	0.9687	0.6901	0.0147

Notes: Means, standard deviations, autocorrelations, correlations with the actual current output, correlations with the output prediction by the true investment wedge, and the RMSE are reported. The RMSE is the root mean squared error of the percentage-deviations between the two output predictions by the true and measured investment wedges.