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An Application of Business Cycle Accounting with Misspecified Wedges*

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Abstract

It is often assumed that wedges evolve according to the VAR (1) in the applications of business cycle accounting (BCA). However, recent research finds that the wedges have no VAR (1) representation in many dynamic stochastic general equilibrium (DSGE) economies, and that there might be a misspecification of the stochastic process of wedges. In order to assess the empirical usefulness of BCA, we apply BCA to a widely used medium-scale DSGE economy. Based on our experiments, we find that the accuracy of the measurement of wedges is high enough to capture the business cycle implications of wedges.

Keywords: business cycle accounting and misspecification.

JEL classification: E10; E32

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1 Introduction

Business cycle accounting (BCA) is a method to investigate the important sources of business cycles. The procedure has two parts. The first part involves measuring distortions using a prototype model with time-varying wedges–which resemble aggregate productivity, labor and investment taxes, and government consumption–such that the prototype model exactly accounts for the observed data. The second part investigates the importance of each wedge in business cycles through counterfactual simulations. BCA has become a popular method of business cycle analysis and has been applied to many countries.[1]

For the theoretical justification of BCA, Chari, Kehoe, and McGrattan (2007a) (hereafter, CKM) show what they called the "equivalence results." A model with frictions, which they called a "detailed model," is equivalent to (covered by) the prototype model if the allocation realized in the detailed model is replicated by the prototype model. For example, a sticky wage model is equivalent to a prototype model with labor wedge. The equivalence holds through an appropriate adjustment of wedges. CKM show the equivalence results without restricting the classes of the stochastic process for wedges.

In many application of BCA, the VAR(1) specification is employed as the stochastic process of wedges. However, recent papers by Bäurle and Burren (2007) and Nutahara and Inaba (2008) show that the equivalence results do not hold in many models if the stochastic process of the wedges of the prototype model is VAR(1). Then, there might be misspecifications of the stochastic process of the wedges in the applications of BCA.[2] The main purpose of the present paper is to quantitatively investigate the significance of the distortions in the measurement of wedges and counterfactual simulations caused by the misspecification of the stochastic process of wedges.

In this paper, in order to assess the empirical usefulness of BCA, we apply BCA to a medium-scale dynamic stochastic general equilibrium (DSGE) model that is not

[1] For example, see the papers by Chari, Kehoe, and McGrattan (2002, 2007a); Ahearne, Kydland, and Wynne (2006); Bridji (2007); Kersting (2008); Kobayashi and Inaba (2006); Otsu (2007); and Saijo (2008).

[2] Christiano and Davis (2006) point out the other problems of BCA.

covered by the prototype model but widely used for policy analyses. Since we know a true data generating process, we can compute the true wedges that are consistent with our DSGE economy. Our economy is based on that of Smets and Wouters (2007) without investment-specific technology shocks. We find that the measured wedges capture the properties of the true wedges almost correctly. We also apply BCA to the economy with investment-specific technology shocks. In this case, the performance of BCA is a little worse. However, the business cycle implications of wedges are captured almost correctly. Therefore, BCA is empirically useful.

The rest of this paper is as follows. Section 2 introduces the prototype economy for BCA and our detailed economy. Section 3 presents our main results. We also apply BCA to our detailed economy in this section. In Section 4, we apply BCA to the economy with investment-specific technology shocks. Section 5 contains the concluding remarks.

2 Basic framework

2.1 Detailed economy: Medium-scale DSGE economy

We employ a medium-scale DSGE model as a laboratory for the assessment of BCA. Since we know a true data generating process, we can compute the true wedges that are consistent with our DSGE economy.

There are two major reasons why we employ a medium-scale DSGE model. One is that the equivalence result does not hold in the medium-scale DSGE model since there are many endogenous and exogenous state variables as shown by Nutahara and Inaba (2008). The other is that such a model has dynamics that are rich enough to produce the actual tendency found in the data, and we already know the empirically plausible range of parameters in such models. Using this model, we investigate the empirical usefulness of BCA under plausible parameter values in a realistic economy.

Our economy is based on that employed by Smets and Wouters (2007). The only difference between our detailed model and the model of Smets and Wouters (2007) is that there are no investment-specific technology shocks in our economy. This is to guarantee that the capital stocks in two economies are the same. For the details of this model and the definition of the true wedges, see Nutahara and Inaba (2010).

2.2 Equivalence results

The definition of the equivalence employed in this paper is as follows.

Definition 1. *A detailed model is equivalent to (covered by) a prototype model if the prototype model can achieve all realized sequences of consumption, investment, labor, output, and capital stock generated in the detailed model.*

CKM give the so-called equivalence results, the prototype model covers a large class of frictional detailed models. However, they do not specify the types of the stochastic process of the wedges that are necessary for the equivalence; note that the VAR(1) specification is often employed when BCA is applied to the actual data.

Bäurle and Burren (2007) and Nutahara and Inaba (2008) investigate the necessary and sufficient condition for the equivalence in the case where the wedges of the prototype model evolve according to the conventional VAR(1) process. They find that in many DSGE economies, the equivalence results do not hold in this case.[3]

The main reason of this is that there is no VAR(1) representation of the true wedges in general. The intuition of Nuahara and Inaba (2008) yields that a VAR(1) representation of the wedges does not exist if the number of endogenous and exogenous state variables in the detailed model is greater than the number of wedges in the prototype model. In particular, the equivalence result does not hold in our medium-scale DSGE economy.

3 Application of BCA

3.1 Procedure of our experiment

First, we generate artificial data in our medium-scale DSGE economy. The parameter values are the same as those estimated by Smets and Wouters (2007) and described in

[3] See Theorem 1 of Nutahara and Inaba (2008) for the necessary and sufficient condition for the equivalence.

Nutahara and Inaba (2010). The only difference is that the steady-state level of labor is set as $\frac{1}{3}$ in our economy. We generate artificial data for 10,000 time periods in order to avoid the small-sample estimation bias.

In the second step, we apply BCA to this artificial data. Our procedure of BCA follows the standard procedure. We set the parameter values of the prototype model as follows: the discount factor, depreciation rate of capital, and share of capital in production are the same as in our detailed model. The parameter for the weight of leisure in the utility is set such that the steady-state labor is $\frac{1}{3}$. There are no adjustment costs of investment in our prototype model.[4] We employ the maximum likelihood method based on the Kalman filter to estimate the VAR(1) parameter for the wedges. The observable variables are consumption, labor, investment, and output. The wedges are measured given the estimated evolution of the wedges. In the estimation, we use the levels of the data. Since this means that we estimate the steady state of the wedges, the steady-state values of consumption, labor, investment, and output in the two economies are the same.

3.2 Main results

Figure 1 shows the true and measured wedges. The solid lines are the true wedges that are consistent with the Smets-Wouters economy. The crosses are the measured wedges given by BCA. The wedges in figures are levels (and not log-deviations). We only show the data of the first 200 periods for viewability.

[Insert Figure 1]

The true wedges are generated such that they are consistent with the Smets-Wouters economy. The efficiency, labor, and investment wedges are measured correctly by the resource constraint, intratemporal conditions, and the aggregate production function of the prototype model. The measurement of the investment wedge is affected by the non-equivalence between the prototype economy and the detailed economy. However, the measured investment wedge appears to be close to the true one.

[4] The adjustment costs of investment do not affect the evolution of capital and the resource constraint in a linearized economy. The adjustment costs only affect the investment wedge since it is in the Euler equation.

Table 1 reports the cyclical behavior of the true and measured investment wedges: means, standard deviations, autocorrelations, correlations with current output, correlations with the true current investment wedge, and the root mean squared error of the percentage-deviations between the true and the measured of investment wedges (RMSE).

[Insert Table 1]

The true investment wedge is slightly more volatile and less persistent than the measured one and the correlation between the true investment wedge and output is slightly larger than that between the measured investment wedge and output. However, the differences are quite small. The correlation between the true and measured investment wedges is close to one. The RMSE gives that the difference between the true and measured investment wedges is almost one percent. Therefore, BCA works well in the measurement of wedges.

In addition to the measurement of wedges, we investigate the output predictions of counterfactual simulations: wedge decompositions. In wedge decompositions, the counterfactual sequences of wedges are constructed as follows. For example, to investigate the contribution of the efficiency wedge, the efficiency wedge is assumed to be the same as the measured efficiency wedge and other wedges are assumed to be constants over time.[5] The misspecification of the stochastic process of wedges might affect the wedge decompositions of *all* wedges, and not only of investment wedge, since the aggregate decision rule depends on the estimated stochastic process of wedges.

Figure 2 shows the output decomposition by each wedge.

[Insert Figure 2]

The dashed-dotted lines are the actual output data. The solid lines are the output predictions by the true wedges, and the bold solid lines are the output predictions by the measured wedges. In our medium-scale DSGE economy, the contributions of the efficiency and labor wedges are significant, and that of the investment wedge is negative

[5] There do exist some decomposition methods and here, we employ the method employed by CKM. We employ the "theoretically-consistent methodology" proposed by Chari, Kehoe, and McGrattan (2007a, 2007b). The decompositions for the true wedges are explained in Nutahara and Inaba (2010).

and small. This is consistent with the findings of CKM during the U.S. Great Depression. It can be easily verified that the output predictions by the true and measured wedges are very close in the cases of efficiency, labor, and government wedges. There are small differences in the case of the investment wedge, but the two predicted outputs positively comove and this would not induce serious misleading implications of the investment wedge on business cycle fluctuations.

Table 2 reports the cyclical behavior of the two output predictions by the true and measured investment wedges.

[Insert Table 2]

The correlation between the two output predictions is 0.7594, which is less than the correlation between the measured investment wedge and the true one. However, the RMSE indicates that the difference between the two output predictions is almost one percent.

As shown by Bäulre and Burren (2007) and Nutahara and Inaba (2008), the equivalence result between the prototype economy and our detailed economy does not hold since the wedges do not have a VAR(1) representation. However, our results in this experiment indicate that the stochastic process of wedges is numerically approximated to the VAR(1) process.

Why is the prototype economy a good approximation of the detailed economy? In order to obtain the underlying reason, we estimate the VAR(1) model using the data of true wedges by the equation-by-equation OLS.[6] The adjusted R^2 values are reported in Table 3. All adjusted R^2 values are close to one, and hence, the fit is good.

[Insert Table 3]

Using the data generated in our detailed model, we also regress the output on the capital stock and the true wedges by OLS. This estimation equation is the policy function in the prototype model. The adjusted R^2 is 0.9988 and very close to one. These results imply that the wedges are approximated by the VAR(1) process and that the output is

[6] We would like to thank the anonymous referee for suggesting the analyses in this paragraph.

approximated by the policy function of the prototype model. Therefore, the prototype economy is a good approximation of our detailed economy.

Finally, we find that, even in our medium-scale DSGE economy, BCA works well.

4 Adding investment-specific technology shocks

In the experiment of Section 3, we assume that there are no investment-specific technology shocks in the detailed economy. Here, we apply BCA to our medium-scale DSGE economy with investment-specific technology shocks.

In the economy with investment-specific technology shocks, the investment Euler equation and the evolution of capital stock change. We assume that the investment-specific technology shock evolves according to an AR(1) process as employed by Smets and Wouters (2007).

Note that capital stock in the detailed economy is no longer the same as that in the prototype economy since the evolution of capital stock in the latter is affected by the investment-specific technology shock. This implies that there is a mismeasurement of capital stock under the prototype model without investment-specific technology shock. In order to focus only on the problem of misspecification of the stochastic process of wedges, we define the true wedge using the capital stock of the prototype economy.[7]

We employ the same parameter value for the AR(1) coefficient of the investment-specific technology shock as estimated by Smets and Wouters (2007). The procedure of BCA is the same as in the baseline economy.

Figure 3 shows the true and measured wedges in our medium-scale DSGE economy with investment-specific technology shocks.

[Insert Figure 3]

With investment-specific technology shocks, the difference between the measured and true investment wedges is still small and other wedges are measured correctly.

Table 4 reports the cyclical behavior of the true and measured investment wedges.

[7] We are grateful to the referee for pointing this out. For the details, see Nutahara and Inaba (2010).

[Insert Table 4]

The correlation between the true and measured investment wedges becomes smaller, and the RMSE indicates that the difference between the two investment wedges is about 1.6 percent. However, the measured investment wedge seems to be close to the true one. At least, the measured investment wedge captures the abstract of the dynamics of the true one.

In the case with investment-specific technology shock, the number of exogenous state variables increases. Generally, the increase in the number of state variables prevents the true wedges from being approximated by the VAR(1) process. Our result implies that the wedges are measured almost correctly even in this case.

Figure 4 shows the output decomposition by each wedge.

[Insert Figure 4]

For efficiency, labor, and investment wedges, the differences between the output predictions of the true and measured wedges are larger in the cases with investment-specific technology shocks. This is because the estimated VAR(1) process of wedges affects the decomposition for all wedges. However, even in this case, the business cycle implications of wedges can be captured by BCA; the contribution of the investment wedge is negative and small.

Table 5 reports the cyclical behavior of the two output predictions by the true and measured investment wedges.

[Insert Table 5]

The RMSE is about 1.2 percent. Finally, we conclude that BCA is empirically useful even in an economy with the investment-specific technology shocks.

5 Conclusion

The premise of BCA is that the prototype model with time-varying wedges can replicate the allocations generated by a large class of models with frictions: the so-called equivalence results. However, some recent papers such as Bäurle and Burren (2007) and Nutahara and Inaba (2008) show that the equivalence results do not hold in many models under the conventional VAR(1) assumption for wedges.

In this paper, in order to investigate the empirical usefulness of BCA, we applied BCA to a medium-scale DSGE economy à la Smets and Wouters (2007). In this economy, the equivalence does not hold. We found that BCA works pretty well in the case without investment-specific technology shocks. In the case with investment-specific technology shocks, the performance of BCA worsens, but the business cycle implications of wedges are captured almost correctly. Our results imply the empirical usefulness of BCA.

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Table 1: Cyclical behavior of the true and measured investment wedges (1): Baseline economy

	mean	std	autocorr.	corr. w/ y_t	corr. w/ true	RMSE
true	1.0016	0.0231	0.9661	-0.3980	–	–
VAR(1)	0.9962	0.0215	0.9803	-0.4002	0.9224	0.0103

Notes: Means, standard deviations, autocorrelations, correlations with the current output, correlations with the true investment wedge, and the RMSE are reported. The RMSE is the root mean squared error of the percentage-deviations between the true and measured investment wedges.

Table 2: Cyclical behavior of the predicted output by the true and measured investment wedges (1): Baseline economy

	mean	std	autocorr.	corr. w/ true	RMSE
true	3.2226	0.0489	0.9330	–	–
VAR(1)	3.2333	0.0366	0.9693	0.7594	0.0105

Notes: Means, standard deviations, autocorrelations, correlations with the actual current output, correlations with the output predicted by the true investment wedge, and the RMSE are reported. The RMSE is the root mean squared error of teh percentage-deviations between the two output predictions by the true and measured investment wedges.

Table 3: Accuracy of the VAR(1) process using the true wedges

equation	efficiency	labor	investment	government
adjusted R^2	0.9509	0.9717	0.9338	0.9472

Notes: The VAR(1) process is by the equation-by-equation OLS. The data are generated in the baseline economy.

Table 4: Cyclical behavior of the true and measured investment wedges (2): With investment-specific technology shock

	mean	std	autocorr.	corr. w/ y_t	corr. w/ true	RMSE
true	1.0018	0.0314	0.9671	-0.1192	–	–
measured	1.0030	0.0234	0.9798	-0.0930	0.8758	0.0158

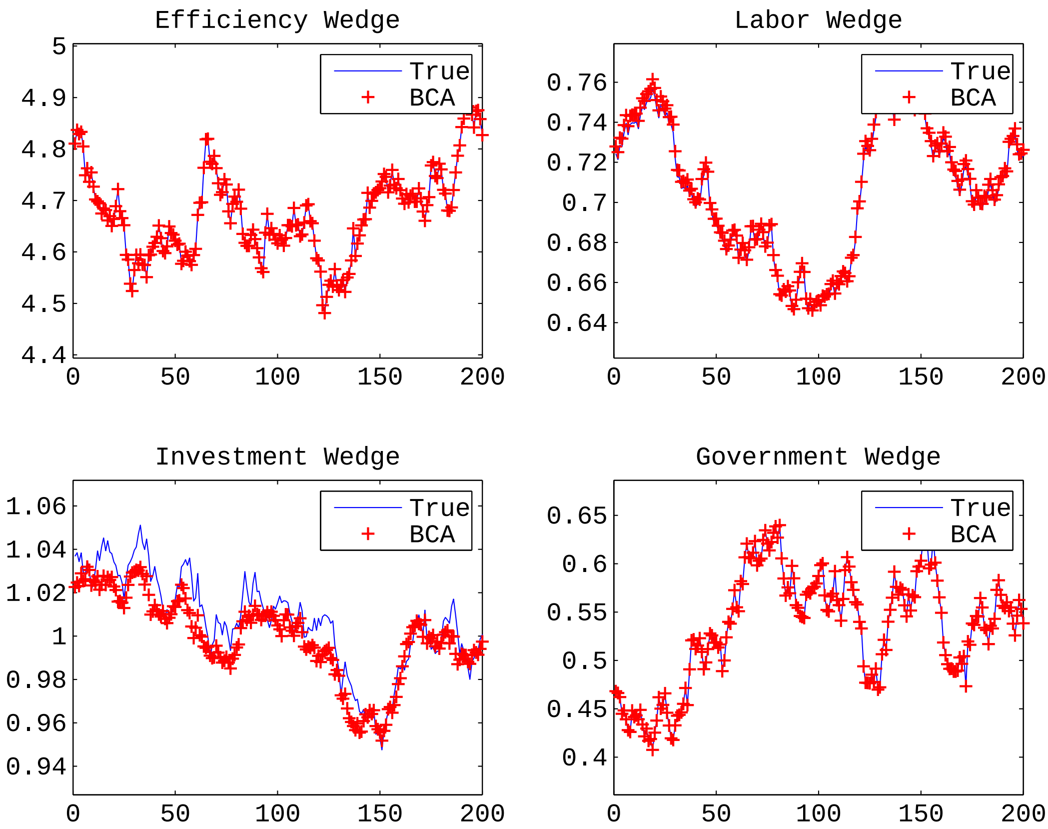
Notes: Means, standard deviations, autocorrelations, correlations with the current output, correlations with the true investment wedge, and the RMSE are reported. The RMSE is the root mean squared error of the percentage-deviations between the true and measured investment wedges.

Table 5: Cyclical behavior of the predicted output by the true and measured investment wedges (2): With investment-specific technology shock

	mean	std	autocorr.	corr. w/ true	RMSE
true	3.2224	0.0548	0.9364	–	–
measured	3.2165	0.0399	0.9692	0.7042	0.0122

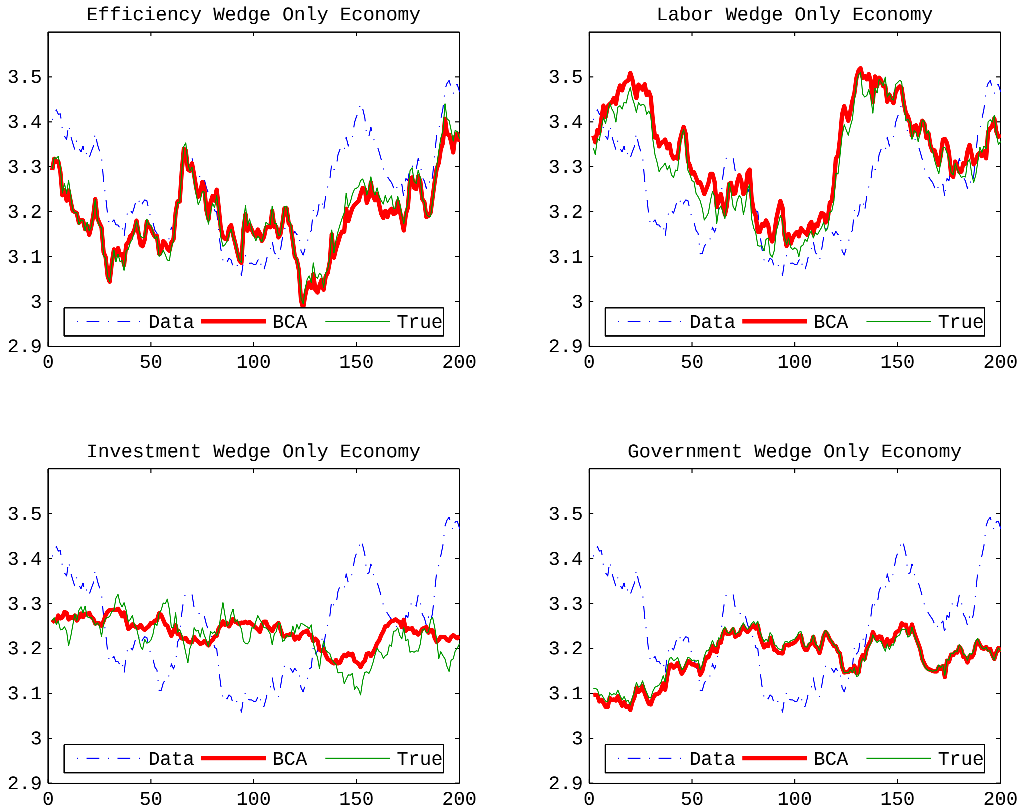
Notes: Means, standard deviations, autocorrelations, correlations with the actual current output, correlations with the output predicted by the true investment wedge, and the RMSE are reported. The RMSE is the root mean squared error of the percentage-deviations between the two output predictions by the true and measured investment wedges.

Figure 1: True and measured wedges (1): Baseline economy



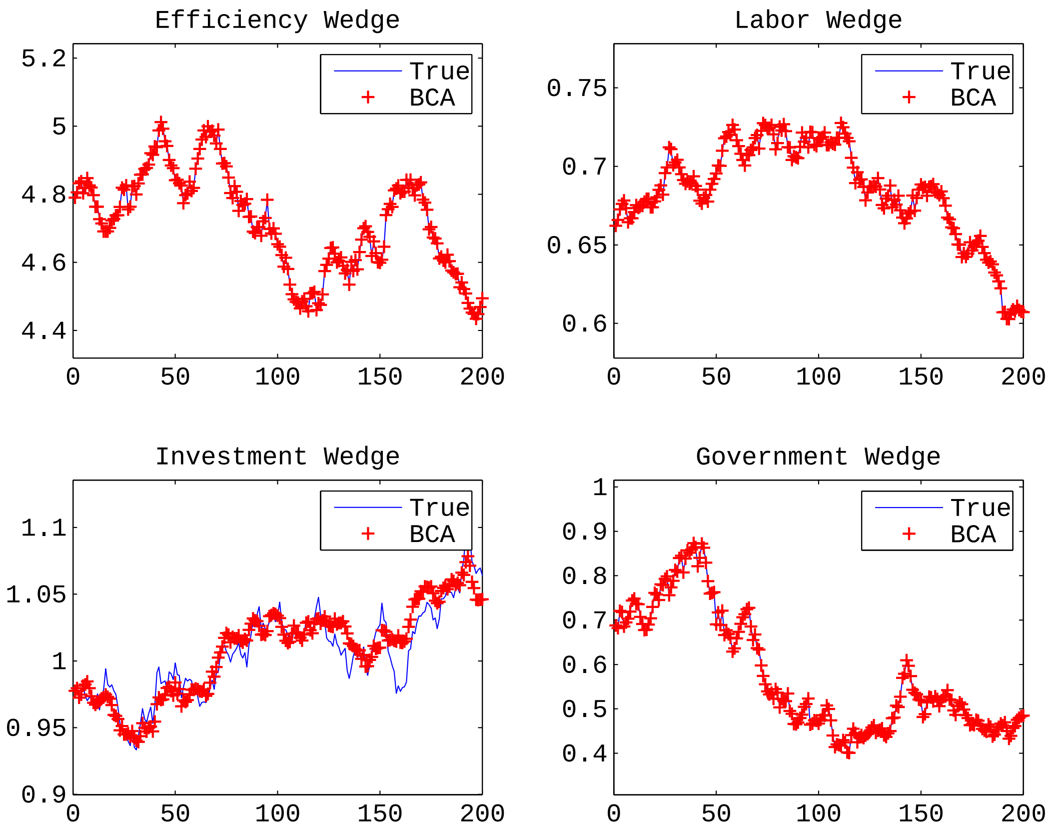
Notes: The solid lines are the true wedges that are consistent with the Smets-Wouters economy. The crosses are the measured wedges by BCA. The wedges in the figures are levels (and not log-deviations).

Figure 2: Output decomposition by the true and measured wedges (1): Baseline economy



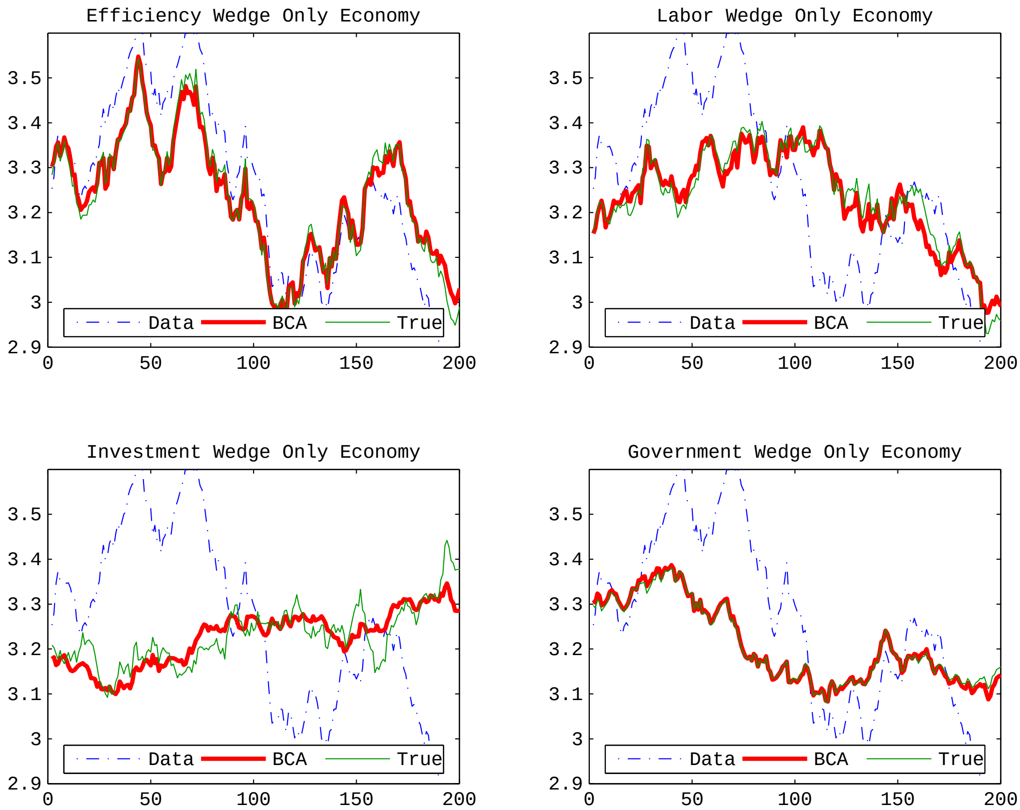
Notes: The dashed-dotted lines are the actual output data. The bold solid lines are the output predictions by the measured wedges. The solid lines are the output predictions by the true wedges.

Figure 3: True and measured wedges (2): With the investment-specific technology shock



Notes: The solid lines are the true wedges that are consistent with the Smets-Wouters economy. The crosses are the measured wedges by BCA. The wedges in the figures are levels (and not log-deviations).

Figure 4: Output decomposition by the true and measured wedge (2): With investment-specific technology shock



Notes: The dashed-dotted lines are the actual output data. The bold solid lines are the output predictions by the measured wedges. The solid lines are the output predictions by the true wedges.