RIETI Discussion Paper Series 09-E-045

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- A model of financial crises -

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Asset-price collapse and market disruption
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March 10, 2009 (First draft: October 30, 2008)

Abstract

We construct a search-theoretic model à la Lagos and Wright (2005), that has multiple steady-state equilibria, one of which may be interpreted as a state of financial crisis. The key ingredient is the collateral-secured loan in the decentralized matching market, in which the borrowers must put up their own land as collateral. They borrow debt for intertemporal smoothing of the consumption stream and also for factor payment in production. In the crisis state, the land price is low and the debt for factor payment, i.e., liquidity, dries up. Facing a liquidity shortage, all sellers choose not to participate in the matching market and the market is shut down due to the search externality. This market disruption lowers the aggregate productivity, while the low productivity justifies the low asset price in turn.

We may be able to derive a policy implication that collective debt reduction by government intervention may solve the coordination failure and bring the economy out of the crisis equilibrium.

Keywords: Collateral-secured loan, search frictions, liquidity, chain of productions.
JEL Classifications: E30, E60, G01, G12.

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I thank Tomoyuki Nakajima and seminar participants at Nihon University, Osaka University (Search Theory Conference), Kyoto University, and Otaru University of Commerce for helpful comments.

1 Introduction

In recent financial crises, asset-price collapses caused tightening of credit in the economy where collateral-secured loans were widespread. Examples are Japan's lost decade in the 1990s, the Asian currency crisis in 1997–1998, and the global financial crisis in 2008. Real estate, such as commercial properties and/or housing assets, is used as collateral for loans all over the world, and once the prices of collateral assets collapse, credit and liquidity are severely tightened.

Macroeconomic theorists so far have proposed two different kinds of models for collateral lending. In one strand of models collateral lending smooths consumption and investment intertemporally (Kiyotaki and Moore 1997; Bernanke, Gertler, and Gilchrist 1999; Iacoviello 2005). In the other models collateral lending is modeled as an intratemporal debt that is used as an instrument for factor payment for production (Carlstrom and Fuerst 1997, 1998; Jermann and Quadrini 2006; Mendoza 2006; Kobayashi, Nakajima and Inaba 2007; Kobayashi and Nutahara 2007). We call the instrument for factor payment *liquidity* in this paper. Collateral lending works either as an intertemporal smoother of consumption and investment or as an intratemporal liquidity in the respective strands of existing models.

In this paper, we propose a model in which collateral lending plays both roles of an intertemporal smoother and of intratemporal liquidity. We show that the interaction between the two roles of collateral lending causes multiple steady-state equilibria to exist. There are two steady states, one of which is a bad equilibrium where the asset price collapses and a decentralized matching market is disrupted due to a shortage of liquidity. In the bad equilibrium, the land price is low and the intratemporal debt, i.e., liquidity, dries up because of the shortage of collateral. Consequently, the decentralized matching market is disrupted: Since patient agents choose whether to participate in the matching market, when the land price is low all patient agents choose not to participate and the market is shut down. The participation choice by patient agents has an external effect on impatient agents through search probability, which makes the bad equilibrium a stable equilibrium. The market disruption lowers the aggregate productivity, while the low productivity justifies the low asset price in turn. We may be able to derive an implication for "macroprudential policy" (Borio 2003, Bernanke 2008) or financial crisis management: Collective debt reduction by impatient agents due to government intervention may solve the coordination failure and bring the economy out of the crisis equilibrium.

Related literature: This paper shows that an asset-price collapse can cause a potentially persistent, maybe decade-long, recession due to a liquidity shortage. The mechanism proposed in this paper may be a possible explanation for persistent productivity declines observed in various episodes of "great depressions." See Kehoe and Prescott (2002, 2007) and references therein for a neoclassical account for the great depressions in the 20th century: In most of the episodes, productivity declines were the primal factor that caused the depressions, while the root causes of productivity declines are left unexplained in these research findings (see also Ohanian 2001). The mechanism of market disruption in our model, that is, the disruption of transactions in the matching market, is close to the disruption of the chains of production (or the division of labor among firms) in Blanchard and Kremer (1997) and Kobayashi (2004, 2006). One contribution of this paper is the model's ability to analyze the relationship between the asset price and the market disruption explicitly through adoption of the search-theoretic framework of Lagos and Wright (2005). Although the Lagos-Wright framework is intended to contribute to monetary theory (see Rocheteau and Wright 2005, and Lagos and Rocheteau 2005, for application of this framework in the monetary theory literature), we use it to construct essentially a "real" model in the spirit of Carlstrom and Fuerst (1997, 1998).

Two-agent example: Before presenting a formal model, we show a simple two-agent example. Suppose that there exist continua of buyers and sellers with measure 1, respectively. The economy continues two periods: $t = 0, 1$. The buyer's utility comes from his consumption at both $t = 0$ and $t = 1$, while the seller's utility comes from his consumption at $t = 1$ only. We assume that the buyer's utility is $b_0 + \beta b_1$, where b_i is his consumption at $t = i$ and that the seller's utility is s_1, which equals his consumption at $t = 1$. We assume that $1/2 < \beta < 1$. Suppose that each buyer is endowed with K units of consumption goods at $t = 0$ and nothing at $t = 1$. The consumption goods can be consumed by both sellers and buyers. Each seller is endowed with nothing at $t = 0$ and 1 unit of intermediate goods at $t = 1$, which can be transformed into $2Q$ units of consumption goods only by a buyer, where $Q < K$. The intermediate goods cannot be consumed by anyone. At $t = 1$, a decentralized matching market opens and all buyers participate in the market, while each seller must pay a cost (i.e., disutility) of κ to enter the market if he decides to participate. In the matching market, buyers and sellers search for their trading partners and matching of a seller and a buyer is made with matching function $\Lambda(1, \mu)$, where 1 is the measure of buyers and μ is the measure of sellers who participate in the matching market. We assume $\Lambda(1, 1) = 1$ and $\Lambda(1, 0) = 0$, that is, the matching occurs with probability 1 if all sellers participate and the matching never occurs if no sellers participate. If the match is made, the seller gives the intermediate good in exchange for the consumption good. We assume for simplicity that an unspecified market institution determines that the price of the intermediate good is Q, that is, the seller gives one unit of the intermediate good in exchange for Q units of consumption goods. At $t = 0$, each buyer chooses consumption, b_0. If $K - b_0 \geq Q$, he can buy the intermediate good and consume $b_1 = K - b_0 + Q$ at $t = 1$. If $K - b_0 < Q$, he cannot buy the intermediate good and consume $b_1 = K - b_0$ at $t = 1$. There are two equilibria for this example. In one equilibrium, no sellers participate in the matching market and the buyers consume K, all their endowment, at $t = 0$ since b_1 is discounted. Therefore, $b_0 = K$ and $b_1 = 0$. The sellers' consumption at $t = 1$ is zero: $s_1 = 0$. In another equilibrium, all sellers participate and get the utility $s_1 - \kappa$, where $s_1 = Q$. The buyers consume $b_0 = K - Q$ at $t = 0$ and $b_1 = 2Q$ at $t = 1$. The welfare is obviously higher in the latter equilibrium than the former. The former corresponds to the financial crisis and the latter corresponds to the normal state in our formal model in the next section.

In the next section, we present the model and show our basic results including policy implication for financial crisis management. In Section 3, we consider a generalized model where multiple matching markets open sequentially in the daytime. Section 4 concludes. In Appendix, we briefly describe a monetary version of our model.

2 Model

The model is a variant of the search-theoretic model developed by Lagos and Wright (2005), in which we introduce collateral lending as a payment instrument just like Ferraris and Watanabe (2008). Unlike Ferraris and Watanabe's model, in which lenders give cash to the borrowers and the total amount of cash is exogenously given by the government, banks in our model can costlessly create the payment instruments, which may be interpreted as bank notes or promissory notes.

2.1 Setup

The model is a closed economy à la Lagos and Wright (2005), in which there are continua of patient agents and impatient agents, who live forever. The measures of patient and impatient agents are M and 1 respectively, while M is sufficiently large:

$$M \gg 1. \tag{1}$$

There is also a unit mass of banks that can create payment instruments (i.e., bank notes) costlessly and lend them to agents as collateral-secured loans. Time is discrete and continues from 0 to infinity: $t = 0, 1, \cdots, \infty$. The numeraire is the consumption good. For each date t, the market is open twice: the day market and the night market. Agents can consume the consumption good only in the night market. The consumption goods are not storable, that is, the consumption goods produced in the day market or the night market of date t must be consumed in the date-t night market, otherwise they perish before date $t+1$ begins. The day market is a decentralized search market in which patient agents and impatient agents trade intermediate goods when they meet. A trade in the day market is *quid pro quo* and no trade credit is available. The night market is a centralized Walrasian market in which patient and impatient agents and banks trade consumption goods, land, bonds issued by impatient agents, and bank notes.

In the day market, when a patient agent and an impatient agent meet each other, the patient agent produces the intermediate good and sells it to the impatient agent at a competitive market price. The impatient agent pays for the intermediate good by giving the bank notes that she borrowed from a bank in the previous night market. Patient agents need to pay a fixed cost for participating in the day market, and they choose whether they participate in the day market or not at the beginning of each period t. In the night market, impatient agents issue intertemporal bonds and patient agents buy the bonds in exchange for the consumption good. Impatient agents also borrow bank notes from banks and can hold productive assets, i.e., land. For simplicity we assume that patient agents cannot hold land. Impatient agents have incomplete commitment technology. Therefore, they cannot precommit to redemption of their bonds nor repayment of their bank loans. However, the impatient agents can use land as collateral for their bonds and bank loans. Banks have complete commitment technology: They can commit themselves to pay one unit of consumption good in the night market in exchange for one unit of bank note that they issued in the previous night market. Therefore, bank notes are circulated as a payment instrument in the day market.

2.2 Optimization problem for patient agents

The state variable for a patient agent who enters the day market is the amount of bonds that she hold from the previous period. She holds bonds issued by impatient agents as her assets. When she enters the night market the bonds and the revenue she earned in the day market are the state variables. We denote the value function for a patient agent when she enters the day market by $V^p(d_{t-1})$ and that for the night market by $W^p(n_t, d_{t-1})$, where d_{t-1} is bond holding and n_t is the revenue (i.e., bank notes) she gets in the day market. Both d_{t-1} and n_t are measured in the unit of the date-t consumption good, that is, one unit of bond or bank note issued in the date-$(t-1)$ night market is a claim to one unit of date-t consumption good.

At the beginning of the day market, a patient agent chooses whether she participate in the day market by paying fixed cost κ for market participation. The Bellman equation for the patient agent is

$$
\begin{aligned}
&V^p(d_{t-1}) \\
&= \max\left\{-\kappa + \alpha(\mu_t)\max_{q_t}[-\gamma(q_t) + W^p(p_t q_t, d_{t-1})] + \{1-\alpha(\mu_t)\}W^p(0, d_{t-1}), \quad W^p(0, d_{t-1})\right\} \qquad (2) \\
&= \max\left\{-\kappa + \alpha(\mu_t)\max_{q_t}[-\gamma(q_t) + W^p(p_t q_t, d_{t-1}) - W^p(0, d_{t-1})], \ 0\right\} + W^p(0, d_{t-1}), \qquad (3)
\end{aligned}
$$

where q_t is the quantity of the intermediate good produced; $\gamma(q_t)$ is the cost for production, which satisfies $\gamma'(q) > 0$, $\gamma''(q) > 0$, and $\gamma(0) = 0$; p_t is the competitive price of the intermediate good; $\alpha(\mu_t)$ is the matching probability for a patient agent with which she meets an impatient agent in the day market, where μ_t $(0 \leq \mu_t \leq M)$ is the measure of the patient agents who participate in the day market. The expected gain for a patient agent from participating in the day market, π, is

$$\pi \equiv -\kappa + \alpha(\mu_t)\max_{q_t}\{-\gamma(q_t) + W^p(p_t q_t, d_{t-1}) - W^p(0, d_{t-1})\}. \qquad (4)$$

Given the aggregate measure of participating patient agents, μ_t, a patient agent decides to participate if $\pi > 0$; not to participate if $\pi < 0$; and is indifferent if $\pi = 0$. The Bellman equation for the night market is

$$W^p(n_t, d_{t-1}) = \max_{c_t, h_t, d_t} c_t - h_t + \beta V^p(d_t), \qquad (5)$$

$$\text{subject to} \quad c_t \leq d_{t-1} - \frac{d_t}{1+r_t} + n_t + h_t, \qquad (6)$$

where c_t is the consumption, h_t is the labor supply, r_t is the real interest rate, and β $(0 < \beta < 1)$ is the intertemporal discount factor. To simplify the analysis, we make the assumption that patient agents directly gain c_t units of utility from consuming c_t, which simplifies the form of collateral constraint in the problem of impatient agents. As is standard in the Lagos-Wright framework, we assume that the labor input in the night market gives linear disutility and is transformed into the consumption good linearly. As shown below, this convention simplifies the analysis greatly by making all agents choose the same value of d_t, and degenerating the heterogeneity among agents with respect to revenues in the period-t day market. Substituting h_t in the budget constraint into (5), we have

$$W^p(n_t, d_{t-1}) = \max_{d_t} d_{t-1} - \frac{d_t}{1+r_t} + n_t + \beta V^p(d_t). \tag{7}$$

The first-order condition (FOC) for (7) is

$$\beta V_d^p(d_t) = \frac{1}{1+r_t}. \tag{8}$$

The envelope conditions for (7) are

$$W_n^p(n_t, d_{t-1}) = 1, \tag{9}$$

$$W_d^p(n_t, d_{t-1}) = 1. \tag{10}$$

These conditions imply that $W^p(n_t, d_{t-1}) = n_t + d_{t-1} + W_0^p$, where $W_0^p \equiv W^p(0,0)$. Therefore, the Bellman equation for the day market can be rewritten as

$$V^p(d_{t-1}) = \max\{-\kappa + \alpha(\mu_t)\max_{q_t}[-\gamma(q_t) + p_t q_t], \quad 0\} + d_{t-1} + W_0^p. \tag{11}$$

Since the price of the intermediate good is determined competitively,[1] FOC for q_t is

$$p_t = \gamma'(q_t). \tag{12}$$

The envelope condition, i.e., $V_d^p(d_{t-1}) = 1$, and (8) imply that

$$\beta(1+r_t) = 1. \tag{13}$$

The measure of the patient agents who participate in the day market, μ_t, is determined endogenously by participation decisions of individual agents. The gain from participation $\pi(\mu_t)$ can be rewritten as

$$\pi(\mu_t) = -\kappa + \alpha(\mu_t)\{-\gamma(q_t) + p_t q_t\}. \tag{14}$$

[1]Although we assume for simplicity price-taking behavior of all agents in the matching market, we can easily modify our model such that the trading scheme in the day market is a bilateral bargaining or competitive search, following Rocheteau and Wright (2005).

There exist multiple equilibria corresponding to different values of μ_t: $\mu_t = 0$ can be an equilibrium since $\pi(0) = -\kappa < 0$ and an impatient agent chooses not to participate when $\pi < 0$; and the solution to $\pi(\mu_t) = 0$ can be an equilibrium, too. Therefore, μ_t is determined by the following equation in equilibrium:

$$\mu_t[\alpha(\mu_t)\{p_t q_t - \gamma(q_t)\} - \kappa] = 0. \tag{15}$$

2.3 Optimization problem for impatient agents

The state variables for an impatient agent who enters the decentralized day market are the amount of bonds she issued in the previous night market (b_{t-1}); the amount of bank loans she borrowed in the previous night market (l_{t-1}); and the land she purchased in the previous night market (k_{t-1}). The impatient agent borrows l_{t-1} from a bank in the form of bank notes, that is, she is given l_{t-1} units of bank notes when she borrows from the bank in the previous night market. Note that b_{t-1} and l_{t-1} are measured in the units of date-t consumption good. The state variables when she enters the current night market are b_{t-1}, l_{t-1}, k_{t-1}, the remaining bank notes (n'_t), and the output she produced in the day market (y_t). In the date-t night market, the impatient agent must repay $R_{t-1}l_{t-1}$ units of the date-t consumption good to the banks, where R_{t-1} is the gross rate of return on the bank loan. The Bellman equation for the day market is

$$\begin{aligned} V(l_{t-1}, b_{t-1}, k_{t-1}) = &\max_{y_t, q_t} \alpha(\mu_t)\mu_t W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) \\ &+ \{1 - \alpha(\mu_t)\mu_t\} W(0, l_{t-1}, l_{t-1}, b_{t-1}, k_{t-1}), \end{aligned} \tag{16}$$

$$\text{subject to} \quad n'_t = l_{t-1} - p_t q_t \tag{17}$$

$$n'_t \geq 0, \tag{18}$$

$$y_t = A k_{t-1}^{\theta} q_t^{1-\theta}, \tag{19}$$

where $V(\,\cdot\,)$ is the value function for an impatient agent who enters the day market, $W(\,\cdot\,)$ is that for those who enters the night market, and $\alpha(\mu_t)\mu_t$ is the matching probability of an impatient agent, meeting a patient agent in the day market.[2]

[2] We assume a standard constant-returns-to-scale matching function, $\Lambda(\mu^p, \mu^i)$, for the day market, where μ^p and μ^i are the measures of patient and impatient agents who participate in the day market. The

The Bellman equation for the night market is

$$W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{c'_t, h'_t, l_t, b_t, k_t} U(c'_t) - h'_t + \beta' V(l_t, b_t, k_t), \tag{20}$$

$$\text{subject to} \quad c'_t + a_t k_t + b_{t-1} + R_{t-1} l_{t-1}$$

$$\leq y_t + h'_t + \omega_t k_{t-1} + n'_t + \frac{b_t}{1+r_t} + a_t k_{t-1}, \tag{21}$$

$$b_t + R_t l_t \leq a_{t+1} k_t, \tag{22}$$

$$l_t \geq 0, \tag{23}$$

where at the beginning of the night market one unit of land generates the dividend ω_t, which is exogenously given by nature; a_t is the date-t land price; c'_t is the consumption; $U(c)$ is the utility from the consumption, where $U'(c) > 0$, $U''(c) < 0$, and $U'(0) = +\infty$; h'_t is the labor supply; and β' is the intertemporal discount factor that satisfies

$$0 < \beta' < \beta < 1. \tag{24}$$

To be more precise, the budget constraint should be written as $c'_t + a_t k_t + b_{t-1} + R_{t-1} l_{t-1} + \frac{l_t}{1+r_t} \leq y_t + h'_t + \omega_t k_{t-1} + n'_t + \frac{b_t}{1+r_t} + a_t k_{t-1} + \frac{l_t}{1+r_t}$, where $\frac{l_t}{1+r_t}$ on the right-hand side (RHS) and that on the left-hand side (LHS) cancel out. The interpretation is as follows: the impatient agent borrows $\frac{l_t}{1+r_t}$ units of date-t consumption goods from the bank, which appears on the RHS, while she immediately purchases l_t units of the bank note, which is just paper but can be used as a means of payment in the date-$(t+1)$ day market, from the bank at price $1/(1+r_t)$; therefore, another $\frac{l_t}{1+r_t}$ appears on the LHS. We also assume the Lagos-Wright convention that h'_t gives linear disutility and is transformed into the consumption good linearly. Because the impatient agent cannot precommit to repay b_t to the creditor (i.e., a patient agent) nor to repay $R_t l_t$ to the bank in the date-$(t+1)$ night market, she must put up her land as collateral for $b_t + R_t l_t$ when she borrows b_t and l_t in the date-t night market. When the impatient agent repudiates her debt, the creditors seize the collateral and sell it off in the date-$(t+1)$ night market. Since there is

matching probability for a patient agent is $\Lambda(\mu^p, \mu^i)/\mu^p = \Lambda(1, \mu^i/\mu^p)$, while that for an impatient agent is $\Lambda(\mu^p, \mu^i)/\mu^i = \Lambda(1, \mu^i/\mu^p)\mu^p/\mu^i$. Since $\mu^p = \mu$ and $\mu^i = 1$ in our model, the matching probability for a patient agent is $\alpha(\mu)$ and that for an impatient agent is $\alpha(\mu)\mu$, where $\alpha(\mu) \equiv \Lambda(1, 1/\mu)$.

no further penalty for repudiation, the debtor surely diverts and defaults on the excess amount of debt that exceeds the value of collateral. Therefore, there is no reason for the creditors to lend more than the value of collateral.[3] The Bellman equation (20) can be rewritten as

$$
\begin{aligned}
&W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) \\
&= \max_{c'_t, l_t, b_t, k_t} U(c'_t) - c'_t + y_t - a_t k_t + (\omega_t + a_t)k_{t-1} - R_{t-1}l_{t-1} + n'_t + \frac{b_t}{1+r_t} - b_{t-1} \\
&\quad + \beta' V(l_t, b_t, k_t),
\end{aligned}
$$

subject to (22) and (23).

The FOCs are

$$U'(c'_t) = 1, \tag{25}$$

$$\frac{1}{1+r_t} - \xi_t + \beta' V_b(l_t, b_t, k_t) = 0, \tag{26}$$

$$\beta' V_k(l_t, b_t, k_t) = a_t - \xi_t a_{t+1}, \tag{27}$$

$$\beta' V_l(l_t, b_t, k_t) + \eta_t - R_t \xi_t = 0, \tag{28}$$

where ξ_t and η_t are the Lagrange multipliers for (22) and (23), respectively. The envelope conditions are

$$
\begin{aligned}
&W_y(t-1) = 1, \qquad W_l(t-1) = -R_{t-1}, \qquad W_{n'}(t-1) = 1, \\
&W_b(t-1) = -1, \qquad W_k(t-1) = \omega_t + a_t,
\end{aligned}
$$

where $W_z(t-1)$ is the derivative of $W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1})$ with respect to z ($= y_t, n'_t, l_{t-1}, b_{t-1},$ or k_{t-1}). These conditions imply that $W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) = y_t -$

[3] Although we do not consider stochastic shocks to the asset prices in this paper, our model can be easily generalized to a stochastic model. In such a case, the collateral constraint, (22), is rewritten as $b_t + R_t l_t \leq E_t[a_{t+1}]k_t$, where $E_t[\,\cdot\,]$ is the mathematical expectations based on the information available at date t. This is the risk neutrality of the patient agent that leads her to evaluate the value of collateral as the mathematical expectation of the value of the asset in the next period. If the utility function of the patient agents is not linear, the collateral constraint in the stochastic model is no longer as simple as the one above.

$R_{t-1}l_{t-1} + n'_t - b_{t-1} + (\omega_t + a_t)k_{t-1} + W_0$, where $W_0 \equiv W(0,0,0,0,0)$. The Bellman equation for the day market can be rewritten as

$$\begin{aligned} V(l_{t-1}, b_{t-1}, k_{t-1}) = &\max_{q_t} \alpha(\mu_t)\mu_t\{Ak_{t-1}^{\theta}q_t^{1-\theta} - p_t q_t\} + (1 - R_{t-1})l_{t-1} - b_{t-1} \\ &+ (\omega_t + a_t)k_{t-1} + W_0, \end{aligned} \tag{29}$$

$$\text{subject to} \quad p_t q_t \leq l_{t-1}, \tag{30}$$

The FOC is

$$p_t = \frac{(1-\theta)Ak_{t-1}^{\theta}q_t^{-\theta}}{1 + x_t}, \tag{31}$$

where x_t is the Lagrange multiplier for (30). The envelope conditions are

$$V_l(l_{t-1}, b_{t-1}, k_{t-1}) = \alpha(\mu_t)\mu_t x_t, \tag{32}$$

$$V_b(l_{t-1}, b_{t-1}, k_{t-1}) = -1, \tag{33}$$

$$V_k(l_{t-1}, b_{t-1}, k_{t-1}) = \omega_t + \alpha(\mu_t)\mu_t\theta Ak_{t-1}^{\theta-1}q_t^{1-\theta} + a_t. \tag{34}$$

Note that the collateral constraint (30) becomes binding with probability $\alpha(\mu_t)\mu_t$ if it binds at all, since it becomes relevant only when the impatient agent successfully matches with a patient agent. Conditions (26)–(28) and (32)–(34) imply the following system of equations that determines the dynamics of the models:

$$\beta'\alpha(\mu_{t+1})\mu_{t+1}x_{t+1} + \eta_t = R_t\xi_t, \tag{35}$$

$$1 = (1 + r_t)\{\xi_t + \beta'\}, \tag{36}$$

$$a_t = \xi_t a_{t+1} + \beta'\left[\omega_{t+1} + \alpha(\mu_{t+1})\mu_{t+1}\theta Ak_t^{\theta-1}q_{t+1}^{1-\theta} + a_{t+1}\right]. \tag{37}$$

2.4 Banks

A bank lends l_t units of bank notes to an impatient agent in the date-t night market. The bank has the commitment technology so that the bank can give one unit of the consumption good in exchange for one unit of the bank note in the date-$(t+1)$ night market. The borrower repays $R_t l_t$ in the form of the consumption good to the bank in the date-$(t+1)$ night market, where R_t is determined competitively. If the borrower repudiates her debt, the bank can seize collateral from the borrower but cannot impose any further penality on the borrower. The bank's profit in units of the date-$(t+1)$ consumption good is $R_t l_t - l_t$. In the date-t night market, the bank chooses l_t to maximize $R_t l_t - l_t$, where there is no restriction on the supply of l_t. Therefore, in equilibrium,

$$R_t = 1. \tag{38}$$

This result parallels that for the intraperiod debt in Carlstrom and Fuerst's (1998) model. Note that $R_t = 1$ is guaranteed by the fact that the bank can create costlessly an unlimited amount of bank notes. If the bank needs to give cash to the borrower and the total supply of cash is fixed exogenously, R_t may exceed 1 when the total amount of cash is small. See Ferraris and Watanabe (2008) for this case.

2.5 Equilibrium

The total supply of land is fixed in this economy:

$$k_t = K. \tag{39}$$

The bond market clears:

$$\forall t, \quad M d_t = b_t. \tag{40}$$

The equilibrium of this economy is defined as follows.

Definition 1 *A competitive equilibrium consists of prices, $\{p_t, R_t, r_t, a_t\}$, and quantities, $\{c_t, c'_t, h_t, h'_t, q_t, k_t, d_t, b_t, l_t, \mu_t\}$, that satisfy that (i) given the prices, the quantities satisfy (12) and (15), which are the optimality conditions for patient agents; (ii) given the prices and $\{\mu_t\}$, quantities solve impatient agents' optimization problems, (20) and (29), and banks' optimization problem; and (iii) equilibrium conditions, (39) and (40), are satisfied.*

The equilibrium path of this economy is described by the prices, $\{p_t, R_t, r_t, a_t, x_t, \xi_t, \eta_t\}$, and the quantities, $\{c'_t, q_t, k_t, b_t, d_t, l_t, \mu_t\}$, where they are the solution to the following system of equations: (12), (13), (15), (22), (25), (30), (31), (35), (36), (37), (38), (39), (40), and

$$\eta_t p_t q_t = 0. \tag{41}$$

Equations (13) and (36) imply that $1 + r_t = 1/\beta$ and $\xi_t = \beta - \beta'$. We can show the following proposition:

Proposition 1 *In equilibrium, either* $(q_t, \mu_t) = (q_h, \mu_h)$ *or* $(q_t, \mu_t) = (0, 0)$*, where* $q_h = q(\mu_h)$ *and* μ_h *is the solution to* $\Pi(\mu) = \kappa$*; where* $\Pi(\mu) \equiv \alpha(\mu)\{\gamma'(q(\mu))q(\mu) - \gamma(q(\mu))\}$ *and* $q(\mu)$ *is an increasing function defined by*

$$\gamma'(q_t)q_t^{\theta} = \frac{\alpha(\mu_t)\mu_t(1-\theta)AK^{\theta}}{\alpha(\mu_t)\mu_t + \frac{\beta}{\beta'} - 1}. \tag{42}$$

Proof: In equilibrium either $\eta_t = 0$ or $\eta_t > 0$. In the case where $\eta_t > 0$, the nonnegativity condition (23) is binding. Since $l_t = p_t q_t = 0$, $q_t = 0$ in equilibrium. Equation (15) implies that since the gain from participating in the day market is zero for a patient agent, all patient agents decide not to participate and $\mu_t = 0$ in this case. Therefore, in equilibrium where $\eta_t = 0$, $(q_t, \mu_t) = (0, 0)$. In the case where $\eta_t = 0$, it must be the case that $\mu_t \neq 0$, since otherwise (35) implies that $\xi_t = 0$, which contradicts the fact that $\xi_t = \beta - \beta' > 0$. Therefore, $\mu_t > 0$ when $\eta_t = 0$. With $\mu_t > 0$, (35) implies that $x_t = (\beta - \beta')/\{\alpha(\mu_t)\mu_t\}$. Therefore, (12), (31), and (39) imply that q_t satisfies (42). Note that (42) determines q_t as an increasing function of μ_t, that is, $q_t = q(\mu_t)$, since the left-hand side of (42) is monotonically increasing in q_t and the right-hand side is monotonically increasing in μ_t. Since $\mu_t > 0$ and $q_t = q(\mu_t)$ in the case where $\eta_t = 0$, (15) implies that μ_t is determined by $\Pi(\mu_t) = \kappa$. Since (42) implies that $q(0) = 0$, $\Pi(0) = 0$. Since the matching probability for a patient agent, $\alpha(\mu)$, converges to zero as $\mu \to \infty$, it must be the case that $\lim_{\mu\to\infty} \Pi(\mu) = 0$. Since $\alpha(\mu)$ is decreasing in μ and $\gamma'(q(\mu))q(\mu) - \gamma(q(\mu))$ is increasing in μ, $\Pi(\mu)$ has only one peak as shown in Figure 1 if functional forms for $\alpha(\,\cdot\,)$ and $\gamma(\,\cdot\,)$ are given appropriately.[4]

Figure 1

[4]For example we can set $\alpha(\mu) = \min\{1, \mu^{-\delta}\}$ and $\gamma(q) = q^{\epsilon}$, where $0 < \delta < 1 < \epsilon$.

We assume that κ is sufficiently small and functional forms for $\alpha(\ \cdot\ )$ and $\gamma(\ \cdot\ )$ are given appropriately such that there exists unique $\overline{\mu}$ such that $\Pi(\mu)$ is increasing in μ for $0 \leq \mu \leq \overline{\mu}$ and decreasing for $\mu \geq \overline{\mu}$. And we assume that κ is sufficiently small such that $\Pi(\mu) = \kappa$ has exactly two solutions, μ_l and μ_h, where $0 < \mu_l < \mu_h < M$. Note that we assumed that the total measure of patient agents, M, is sufficiently large. $\Pi(\mu)$ is increasing in the neighborhood of μ_l and decreasing in the neighborhood of μ_h. The equilibrium with $\mu = \mu_l$ is unstable because a slight deviation from μ_l is amplified by participation decisions of individual agents. The equilibrium with μ_h is stable, since a deviation from μ_h is corrected by participation decisions by individual agents. So we can focus on the solution $\mu_t = \mu_h$. We have shown that for appropriate parameters and functional forms, $(q_t, \mu_t) = (q_h, \mu_h)$ in equilibrium where $\eta_t = 0$. *Q.E.D.*

Since there is no technological innovation nor capital accumulation in this model, there are no time-varying state variables relevant to the aggregate dynamics of the economy. So any sequence of $\{q_t, \mu_t\}_{t=0}^{\infty}$ that satisfies $\forall t, (q_t, \mu_t) \in \{(q_h, \mu_h), (0, 0)\}$, can be an equilibrium path: Given any such sequence of $\{q_t, \mu_t\}_{t=0}^{\infty}$, the sequence of $\{a_t, b_t\}_{t=0}^{\infty}$ is determined by (37) and

$$b_t = a_{t+1}K - p_h q_h \frac{\mu_{t+1}}{\mu_h}, \tag{43}$$

where $p_h = \gamma'(q_h)$. Therefore, an equilibrium path of this model is a sunspot equilibrium in the sense that any path that satisfies Proposition 1 can be the equilibrium outcome solely depending on the agents' expectations.

2.6 Steady States

In what follows in this paper we focus on the steady-state equilibria. Steady-state values of the asset price, a, and bonds, b, are determined by

$$a = \frac{\beta'}{1-\beta}\{\omega + \alpha(\mu)\mu\theta AK^{\theta-1}q^{1-\theta}\}, \tag{44}$$

$$b + pq \leq aK, \tag{45}$$

$$\tag{46}$$

The welfare of patient and impatient agents is measured by the values of their value functions at the beginning of the daytime in the steady state:

$$V^p = d = \frac{b}{M}, \tag{47}$$

$$V = \frac{\alpha(\mu)\mu}{1-\beta'}\{AK^\theta q^{1-\theta} - pq\} + \frac{\omega K}{1-\beta'} - \frac{1-\beta}{1-\beta'}b, \tag{48}$$

where V^p and V are the welfare of patient and impatient agents, respectively. If we define the total welfare of the economy in the steady state, E, as the sum of the patient and impatient agents' welfare, it can be written as

$$E \equiv MV^p + V = \frac{\alpha(\mu)\mu}{1-\beta'}\{AK^\theta q^{1-\theta} - pq\} + \frac{\omega K}{1-\beta'} + \frac{\beta-\beta'}{1-\beta'}b. \tag{49}$$

There exist two types of steady state in our model: one with $(q, \mu) = (q_h, \mu_h)$ and the other with $(q, \mu) = (0, 0)$.

2.6.1 Normal state

We call steady state with $(q, \mu) = (q_h, \mu_h)$ the "normal state." The asset price and the amount of bond are determined by

$$a_n = \frac{\beta'\{\omega + \alpha(\mu_h)\mu_h \theta AK^{\theta-1} q_h^{1-\theta}\}}{1-\beta}, \tag{50}$$

$$b_n = a_n K - p_h q_h, \tag{51}$$

where $q_h = q(\mu_h)$ and $p_h = \gamma'(q_h)$. In the equilibrium where μ is positive, collateral is used to issue intertemporal bonds for consumption smoothing and also it is used to borrow bank notes for factor payment in the decentralized day market. The day market in which agents form chains of productions is operative in the equilibrium where $\mu > 0$, while it is shut down in the equilibrium we describe in the following section.

2.6.2 Crisis state

We call the steady state with $(q, \mu) = (0, 0)$ the "crisis state," because the decentralized day market is shut down in this steady state. The asset price and bonds are determined by

$$a_c = \frac{\beta' \omega}{1-\beta}, \tag{52}$$

$$b_c = a_c K. \tag{53}$$

Obviously from (50) and (52), the asset price is lower in the crisis state than in the normal state: $a_n > a_c$. The debt-asset ratio, $b/(aK)$, is higher in the crisis state than in the normal state:

$$\frac{b_n}{a_n K} = \frac{1}{a_n K}(a_n K - p_h q_h) < 1 = \frac{b_c}{a_c K}. \tag{54}$$

It is shown as follows that the total welfare in the normal state, E_n, is higher than that in the crisis state, E_c. Define $\Delta \equiv (1-\beta')(E_n - E_c)$. Then (49) implies

$$\Delta = \alpha(\mu_h)\mu_h\{AK^\theta q_h^{1-\theta} - \gamma'(q_h)q_h\} + (\beta - \beta')\left\{\frac{\beta'}{1-\beta}\alpha(\mu_h)\mu_h\theta AK^\theta q_h^{1-\theta} - \gamma'(q_h)q_h\right\}. \tag{55}$$

Since (42) implies $\gamma'(q)q = (1-\theta)\beta'\alpha(\mu)\mu AK^\theta q^{1-\theta}/\{\beta'\alpha(\mu)\mu + \beta - \beta'\}$ and $\{\alpha(\mu)\mu + \beta - \beta'\}/\{\beta'\alpha(\mu)\mu + \beta - \beta'\} < 1/\beta'$, this equation implies

$$\Delta > \left\{1 + \frac{(\beta - \beta')\beta'}{1-\beta}\right\}\theta\alpha(\mu_h)\mu_h AK^\theta q_h^{1-\theta} > 0. \tag{56}$$

Therefore, the total welfare, E, is higher in the normal state than in the crisis state.

2.7 Coordination failure in the crisis state and policy implications

In the date-t night market, impatient agents choose l_t, given μ_{t+1}; and at the beginning of date-$(t+1)$, the measure of participating patient agents, μ_{t+1}, is determined by participation choices of individual patient agents, which are made taking μ_{t+1} and l_t as given.

In the crisis state, patient agents collectively set $\mu_{t+1} = 0$ and impatient agents choose $l_t = 0$ for all t. Given that $l_t = 0$, a patient agent has no incentive to participate in the date-$(t+1)$ day market: If she participates, she can meet with an impatient agent with probability 1 because $\mu_{t+1} = 0$. (Note that all impatient agents participate in the day market.) But the impatient agent has no payment instrument, and therefore the patient agent can get nothing in the day market, while she must pay κ when she enters the day market. Thus the expected gain from participation is negative for a patient agent.

Given that $\mu_{t+1} = 0$, an impatient agent has no incentive to borrow from a bank in the date-t night market and bring bank notes into the date-$(t+1)$ day market: Since she can meet with a patient agent in the day market with zero probability, the bank notes are useless. Meanwhile, she wants to maximize the amount of bonds that she issues, b_t, since the market rate, $r_t = \beta^{-1} - 1$ is cheaper than her subjective rate of time discount, i.e., $(\beta')^{-1} - 1$. Therefore, all impatient agents set b_t at its maximum possible value, $a_{t+1}K$, and they set $l_t = 0$.

Note that the coordination failure that causes the crisis equilibrium to exist occurs due to the existence of both the intertemporal consumption loan, b_t, and the loan for intratemporal payment, l_t. Both debts are necessary to make multiple equilibria. For example, if we did not introduce consumption loan, b_t, in our model, the crisis state would be eliminated and the normal state would become the only steady-state equilibrium because l_t is always positive under the collateral constraint, $l_t \leq a_{t+1}k_t$. The novel feature of our model, that is, the use of the collateral-secured loan as both intertemporal consumption loan and intratemporal liquidity, enables the multiple steady-state equilibria to exist.

Policy implications: As we saw above, there is no incentive for impatient agents to reduce their bonds individually in the crisis state. Therefore, the effectiveness of a government intervention may be what is necessary to coordinate the expectations and eliminate the crisis equilibrium. For example, if the government intervenes and imposes the restriction that all impatient agents must set their intertemporal liabilities, b_t, such that $b_t \leq a_{t+1}k_t - p_h q_h$, then the crisis state is eliminated and the economy can immediately jump to the normal state. The government-coordinated debt restriction may be interpreted as a simplified model of the government policies during the episodes of financial crises, such as the \$ 700 billion TARP (Troubled Assets Relief Program) scheme initiated by the Treasury in 2008, in which the US government will buy and dispose of the nonperforming assets and will ultimately bail out debt-ridden households and firms. In past episodes of financial crises all over the world, the governments of the crisis-affected countries undertook various crisis-management policies to urge financial institutions to dispose of nonperforming assets and to reduce the debt burdens in the private sector. In light of our model, we can consider that the essence of these crisis-management policies may be to restrict b_t in order to restore the supply of liquidity, l_t.

Another policy implication would be that if the binding collateral constraint for borrowers is crucial in a financial crisis, policies targeted at lenders may not be effective enough to attain economic recovery. For example, rehabilitation of the banking sector by capital injections and liquidity provision to banks by the central bank may not effectively resolve market disruptions due to a financial crisis unless the debt reduction of borrowers is properly addressed.

3 Generalization – Chains of production

There may be several directions for generalization of this model. One of the most intriguing generalizations would be to incorporate fiat money into this model in a meaningful way, while it turns out to be impossible in our patient-and-impatient-agents framework. Because of the difference in time discount factors, impatient agents never hold nominal money in a steady-state equilibrium. It is also easily shown that even when an interest-bearing money, i.e., bank deposits, is introduced, the impatient agents do not hold deposit money in a steady state (See Appendix).

In this section, we consider a generalization that makes our stylized model a little closer to the reality of chains of production and business cycles by introducing multiple matching markets which open distinct subperiods in the daytime. We can assume that impatient agents go through the matching markets sequentially during the daytime, while in each market they produce the consumption good using land and the intermediate good produced by patient agents. We assume a simple structure of chains of production and an impatient agent, who could not produce in a matching market, cannot enter the subsequent matching markets and must go to the night market directly; and only an impatient agent who successfuly produces in a matching market can enter the next matching market. In equilibrium, several matching markets may be operative, while the other matching markets are shutdown. Although we have only two steady states in the basic model, there arise naturally more than two steady-state equilibria in the generalized model, each of which is distinguished by the number of open matching markets or, equivalently, by the level of land prices. In a steady state with a higher land price, more matching markets are operative. The steady states in the generalized model may be interpreted as representing various stages in the business cycle, such as a boom, a shallow recession, a deep recession, etc. Thus the generalized model may be potentially useful to analyze both ordinary business cycles and extraordinary financial crises in a unified framework.

The generalized model shows a new theoretical possibility of expectations-driven business cycles: Changes in asset prices may cause changes in the current and future productivity through changes in the expectations of the number of operative matching markets, while the productivity changes justify the fluctuations in the asset prices. In short, the generalized model shows that the expectations of the operative matching markets may drive the business cycles, while the expectations are justified by the induced productivity changes.[5] The features of our model are exactly the same as sunspot equilibrium models in that the number of operative matching markets, the asset price, and the aggregate productivity are endogenously selected from candidates of steady states, and there is no selection mechanism that is based on economic fundamentals.

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[5]This type of the expectations-driven business cycle may be interesting because in the existing models of the expectations-driven business cycles pioneered by Beaudry and Portier (2004), the expectations on future productivity is exogenously provided, while in our model the expectations change endogenously. For examples of the existing models of the expectations-driven business cycles, see Christiano, Ilut, Motto and Rostagno (2007), Jaimovich and Rebelo (2008), and Kobayashi, Nakajima, and Inaba (2007).

3.1 Setup

The model is the same as the basic model described in the previous section except that there are N subperiods in the daytime, where $N \geq 2$, and a distinct matching market opens in each subperiod. We call the matching market in the i-th subperiod the D-i market, where $i = 1, 2, \cdots, N$.

3.2 Optimization of impatient agents

An impatient agent enters the D-1 market in the first subperiod of the daytime of date t, bringing l_{t-1} units of bank notes that are borrowed from a bank, b_{t-1} units of debt obligations, and k_{t-1} units of land from the date-$(t-1)$ night market. The impatient agent searches for a patient agent, and if she successfully meets a patient agent she buys the intermediate good from the patient agent and produces the consumption good; and only if she successfully produces in the D-1 market does she enter the D-2 market in the second subperiod, bringing the output of the D-1 market and the remaining bank notes. If she does not meet a patient agent in D-1 market, she cannot participate in any subsequent markets in the daytime and must go directly to the night market. This process is repeated N times, going from D-1 to D-N markets. In the D-i market, an impatient agent meets a patient agent with probability $\alpha(\mu_t^{(i)})\mu_t^{(i)}$, where $\mu_t^{(i)}$ is the *relative* measure of patient agents who participate in the D-i market, that is, the measure of participating patient agents divided by the measure of impatient agents who participate in the D-i market.[6] We denote variables in the D-i market with the superscript (i).

The Bellman equation for the D-i market ($1 \leq i \leq N$) is as follows. The state variables in the value function are $y_t^{(i-1)}$, the consumption good that the agent carries at the beginning of the D-i market; $n_t^{(i-1)}$, the remaining bank notes that the agent carries at the beginning of the D-i market; l_{t-1}, the bank loan that the agent borrowed in the

[6] Using the relative measure, $\mu_t^{(i)}$, the measure of impatient agents who participate in the D-i market is written as $\prod_{j=1}^{i-1} \alpha(\mu_t^{(j)})\mu_t^{(j)}$ and the measure of patient agents who participate in the D-i market is $[\prod_{j=1}^{i-1} \alpha(\mu_t^{(j)})\mu_t^{(j)}]\mu_t^{(i)}$.

date-$(t-1)$ night market; b_{t-1}, the bond; and k_{t-1}, the land.

$$V^{(i)}(y_t^{(i-1)}, n_t^{(i-1)}, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{y^{(i)}, q^{(i)}, n^{(i)}} \alpha(\mu_t^{(i)})\mu_t^{(i)} V^{(i+1)}(y_t^{(i)}, n_t^{(i)}, l_{t-1}, b_{t-1}, k_{t-1})$$
$$+ \{1 - \alpha(\mu_t^{(i)})\mu_t^{(i)}\} W(y_t^{(i-1)}, n_t^{(i-1)}, l_{t-1}, b_{t-1}, k_{t-1}), \tag{57}$$

$$\text{subject to } n_t^{(i)} = n_t^{(i-1)} - p_t^{(i)} q_t^{(i)}, \tag{58}$$

$$n^{(i)} \geq 0, \tag{59}$$

$$y_t^{(i)} = A^{(i)}\{k_{t-1}\}^{\theta}\{q_t^{(i)}\}^{1-\theta} + y_t^{(i-1)}, \tag{60}$$

where $V^{(i)}(\ \cdot\ )$ is the value function for an impatient agent entering the D-i market; $V^{(N+1)}(\ \cdot\ ) \equiv W(\ \cdot\ )$; $y_t^{(0)} = 0$; and $n_t^{(0)} = l_{t-1}$. Note that the production technology given in (60) guarantees that the production in the D-i market is independent from the output in the D-j market for $i \neq j$. Note also that the assumption of chains of production that only those who successfully produce in the D-i market can enter the D-$(i+1)$ market greatly simplifies the analysis by avoiding the curse of dimensionality. This is because this assumption guarantees that all impatient agents who participate in a matching market have identical trading history and have identical values of state variables.

Since the problem for an impatient agent in the night market is identical to the basic model, the Bellman equation for the night market is

$$W(y_t, n_t, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{c_t', h_t', n_{t+1}^{(0)}, l_t, b_t, k_t} U(c_t') - h_t' + \beta' V^{(1)}(0, n_{t+1}^{(0)}, l_t, b_t, k_t), \tag{61}$$

$$\text{subject to} \quad c_t' + a_t k_t + b_{t-1}$$
$$\leq y_t + h_t' + \omega_t k_{t-1} + n_t - R_{t-1} l_{t-1} + \frac{b_t}{1+r_t} + a_t k_{t-1}, \tag{62}$$

$$b_t + R_t l_t \leq a_{t+1} k_t, \tag{63}$$

$$l_t \geq 0, \tag{64}$$

$$n_{t+1}^{(0)} \leq l_t. \tag{65}$$

The same arguments as those in the previous section imply that the value function for the night market can be written as

$$W(y_t, n_t, l_{t-1}, b_{t-1}, k_{t-1}) = y_t + n_t - R_{t-1}l_{t-1} - b_{t-1} + (\omega_t + a_t)k_{t-1} + W_0. \tag{66}$$

Using this expression, the reduced form of the value function $V^{(1)}(\,\cdot\,)$ is derived, given $\{p_t^{(i)}, \mu_t^{(i)}\}_{i=1}^N$: First, $V^{(N)}(\,\cdot\,)$ is solved using (66); and $V^{(i)}(\,\cdot\,)$ is solved backwardly from $i = N-1$ to $i = 1$. Thus we have

$$V^{(1)}(0, n_t^{(0)}, l_{t-1}, b_{t-1}, k_{t-1}) = \sum_{i=1}^N \max_{q_t^{(i)}, n_t^{(i)}} \left[\prod_{j=1}^i \alpha(\mu_t^{(j)})\mu_t^{(j)}\right] \left[A^{(i)} k_{t-1}^\theta \{q_t^{(i)}\}^{1-\theta} - p_t^{(i)} q_t^{(i)}\right]$$

$$- R_{t-1}l_{t-1} - b_{t-1} + (\omega_t + a_t)k_{t-1} + W_0, \tag{67}$$

$$\text{subject to} \qquad p_t^{(i)} q_t^{(i)} + n_t^{(i)} \leq n_t^{(i-1)}, \qquad \text{for } i = 1, 2, \cdots, N, \tag{68}$$

$$n^{(i)} \geq 0, \qquad \text{for } i = 1, 2, \cdots, N. \tag{69}$$

Given $\{p_t^{(i)}, \mu_t^{(i)}\}_{i=1}^N$, the FOCs are

$$p_t^{(i)} = \frac{(1-\theta)A^{(i)} k_{t-1}^\theta \{q_t^{(i)}\}^{-\theta}}{1 + x_t^{(i)}}, \tag{70}$$

$$x_t^{(i-1)} = \beta' \alpha(\mu_t^{(i)})\mu_t^{(i)} x_t^{(i)} + \eta_t^{(i-1)}, \qquad \text{for } i = 1, 2, \cdots, N, \tag{71}$$

where $x_t^{(i)}$ and $\eta_t^{(i)}$ are the Lagrange multipliers for (68) and (69), respectively, and the FOCs for (61) and the envelope conditions for (67) imply that $x_t^{(0)} = \beta - \beta'$ and $\eta_t^{(0)} = 0$.

3.3 Optimization of the patient agent

We assume that at the beginning of each date a patient agent chooses whether she participates in a decentralized matching market in the daytime. A patient agent can participate in at most one market among D-1, $\cdots$, D-N markets. Thus the optimization problem for a patient agent is identical to the basic model. Therefore, $\mu_t^{(i)}$, the relative measure of patient agents who participate in the D-i market, is determined by

$$\left[\alpha(\mu_t^{(i)})\{p_t^{(i)} q_t^{(i)} - \gamma(q_t^{(i)})\} - \kappa\right] \mu_t^{(i)} = 0. \tag{72}$$

If $\mu_t^{(i)} > 0$, $q_t^{(i)}$ is determined by the FOC:

$$\gamma'(q_t^{(i)}) = p_t^{(i)}. \tag{73}$$

3.4 Equilibrium

We can define a competitive equilibrium of the N-market model as follows.

Definition 2 *A competitive equilibrium consists of prices, $\{\{p_t^{(i)}\}_{i=1}^N, R_t, r_t, a_t\}$, and quantities, $\{c_t, c_t', h_t, h_t', k_t, d_t, b_t, l_t, \{q_t^{(i)}, n_t^{(i)}, \mu_t^{(i)}\}_{i=1}^N\}$, that satisfy that (i) given the prices, the quantities satisfy (72) and (73), which are the optimality conditions for patient agents; (ii) given the prices and $\{\mu_t^{(i)}\}_{i=1}^N$, the quantities solve impatient agents' optimization problems, (67) and (61), and banks' optimization problem; and (iii) equilibrium conditions, (39) and (40), are satisfied.*

The equilibrium values of variables in the D-i market are determined by solving (70), (71), (72) and (73) forwardly, given $x_t^{(0)} = \beta - \beta'$ and $\eta_t^{(0)} = 0$, where $\eta_t^{(i)} = 0$ if the D-i market is operative. There exist multiple steady-state equilibria. The D-i market is either open or shutdown for $i = 1, 2, \cdots, N$ in a steady-state equilibrium. Since the assumption of chains of production guarantees that if $\mu_t^{(i)} = 0$ then $\mu_t^{(j)} = 0$ for all $j \geq i + 1$, there exist at most $N + 1$ steady-state equilibria: Each equilibrium is distinguished by the number of the matching markets that are not shutdown. We can define I-equilibrium ($I = 0, 1, \cdots, N$) as a steady-state equilibrium where the first I matching markets are operative, that is, $\mu^{(i)} > 0$ for $1 \leq i \leq I$ and $\mu^{(i)} = 0$ for $I+1 \leq i \leq N$. In I-equilibrium, the equilibrium variables in the D-i market are determined by the following system of equations, given $x^{(0)} = \beta - \beta'$:

$$p^{(i)} = \frac{(1-\theta)A^{(i)} k_{t-1}^{\theta} \{q^{(i)}\}^{-\theta}}{1 + x^{(i)}}, \tag{74}$$

$$x^{(i-1)} = \beta' \alpha(\mu^{(i)}) \mu^{(i)} x^{(i)}, \tag{75}$$

$$\alpha(\mu^{(i)}) \{p^{(i)} q^{(i)} - \gamma(q^{(i)})\} - \kappa = 0, \tag{76}$$

$$\gamma'(q^{(i)}) = p^{(i)}, \tag{77}$$

for $i = 1, 2, \cdots, I$; and $q^{(i)} = \mu^{(i)} = 0$ for $i = I + 1, \cdots, N$.

The values of $\{p^{(i)}, q^{(i)}, \mu^{(i)}, x^{(i)}\}_{i=1}^N$ are determined by solving the system of equations (74)–(77) forwardly from $i = 1$ to $i = N$, given $x^{(0)} = \beta - \beta'$. Note that if the D-i market is open in both I-equilibrium and I'-equilibrium, the values of variables in the D-i market are identical in I- and I'-equilibria for $I \neq I'$.[7]

The asset price in I-equilibrium is determined by

$$a(I) = \frac{\beta' \{\omega + \sum_{i=1}^{I} [\prod_{j=1}^{i} \alpha(\mu^{(j)}) \mu^{(j)}] \theta A^{(i)} K^{\theta-1} \{q^{(i)}\}^{1-\theta}\}}{1-\beta}, \tag{78}$$

or, equivalently, $a(I) = a(I-1) + \frac{\beta'}{1-\beta}[\prod_{j=1}^{I} \alpha(\mu^{(j)})\mu^{(j)}]\theta A^{(I)} K^{\theta-1}\{q^{(I)}\}^{1-\theta}$. Obviously, $a(I)$ is increasing in I, that is, the asset price is higher in the equilibrium where more matching markets are operative. The amount of bonds in I-equilibrium is determined by

$$b(I) = a(I)K - \sum_{i=1}^{I} p^{(i)} q^{(i)}. \tag{79}$$

The welfare of an agent can be defined as the value of her value function at the beginning of the daytime in the steady state. Thus the welfare of a patient agent in I-equilibrium, $V^p(I)$, and that of an impatient agent, $V^{(1)}(I)$, are

$$V^p(I) = d(I) = \frac{b(I)}{M}, \tag{80}$$

$$V^{(1)}(I) = \frac{1}{1-\beta'} \left[\omega K - (1-\beta) b(I) + \sum_{i=1}^{I} \left\{ \prod_{j=1}^{i} \alpha(\mu^{(j)}) \mu^{(j)} \right\} \left(A^{(i)} K^{\theta} \{q^{(i)}\}^{1-\theta} - p^{(i)} q^{(i)} \right) \right]. \tag{81}$$

We can define the total welfare of the economy in I-equilibrium by $E(I) \equiv MV^p(I) + V^{(1)}(I)$, which is written as

$$E(I) = \frac{1}{1-\beta'} \left[\omega K + (\beta - \beta') b(I) + \sum_{i=1}^{I} \left\{ \prod_{j=1}^{i} \alpha(\mu^{(j)}) \mu^{(j)} \right\} \left(A^{(i)} K^{\theta} \{q^{(i)}\}^{1-\theta} - p^{(i)} q^{(i)} \right) \right]. \tag{82}$$

[7] Note also that for a certain range of parameter values there may exist $J(\leq N)$ such that the above system of equations (74)–(77) has no positive solution $\{p^{(i)}, q^{(i)}, \mu^{(i)}, x^{(i)}\}$ for $i = J$. In this case, D-J, D-$(J+1)$, $\cdots$, D-N markets are always shutdown in any equilibrium. In this case, there exist only J steady-state equilibria: 0-, 1-, $\cdots$, $(J-1)$-equilibria.

It is shown, as follows, that $E(I)$ is increasing in I, that is, the welfare of the economy increases as the number of operative matching markets increases. Define $\Delta(I) \equiv (1 - \beta')\{E(I) - E(I-1)\}$.

$$\Delta(I) = \left[\left\{1 + \frac{(\beta - \beta')\beta'\theta}{1-\beta}\right\}\Gamma(I) - \{\Gamma(I) + \beta - \beta'\}\frac{1-\theta}{1+x^{(I)}}\right] A^{(I)}K^{\theta}\{q^{(I)}\}^{1-\theta}, \quad (83)$$

where $\Gamma(I) = \prod_{i=1}^{I}\alpha(\mu^{(i)})\mu^{(i)}$. Since $x^{(I)} = (\beta - \beta')/[(\beta')^I\Gamma(I)]$ and $\{\Gamma(I) + \beta - \beta'\}(\beta')^I/\{(\beta')^I\Gamma(I) + \beta - \beta'\} < 1$, it is shown that

$$\Delta(I) > \left\{1 + \frac{(\beta - \beta')\beta'}{1-\beta}\right\}\theta\Gamma(I)A^{(I)}K^{\theta}\{q^{(I)}\}^{1-\theta} > 0, \quad (84)$$

which implies that $E(I) > E(I-1)$ for all $I \geq 1$.

Policy implications: Multiple steady-state equilibria arise in this generalized model due to the same coordination failure that we discussed in the previous section. This model is essentially a sunspot equilibrium model and the equilibrium selection depends on the macroeconomic expectations. Government policy or regulation that restricts the amount of debt issued by impatient agents may improve the coordination failure and may change the equilibrium. For example, if the government imposes the restriction that $b_t \leq a_{t+1}k_t - \sum_{i=1}^{J} p^{(i)}q^{(i)}$, then the candidates of the steady-state equilibrium that can be realized become restricted to J-, $(J+1)$-, $\cdots$, N-equilibria. In other words, the steady states from 0-equilibrium to $(J-1)$-equilibrium are eliminated from the candidates of realizable steady-state equilibrium. Therefore, this model implies that monetary policy and/or financial regulations that restrict the aggregate level of debt may raise the aggregate productivity and asset prices through increasing liquidity, which enhances economic transactions in decentralized matching markets. This policy implication may be consistent with the historical episodes of financial crises. Japan and Sweden, for example, experienced collapses of land prices and the emergence of nonperforming loans problems almost simultaneously in the early 1990s. Sweden disposed of bad loans aggressively in 1992–1994, and then attained a V-shaped economic recovery in the middle of the 1990s, while Japan postponed the disposal of nonperforming loans and its debt-ridden economy experienced slow growth that lasted a decade.

4 Conclusion

In this paper, we propose a model in which a collateral-secured loan is used to smooth consumption intertemporally and also used as a payment instrument, i.e., liquidity, intratemporally. We show that the interaction between the two roles of collateral lending causes multiple steady-state equilibria to exist. The role of liquidity to mitigate the search friction in the decentralized market plays the key role to generate the multiple equilibria. In our basic model, there are two stable steady states, one of which is a bad equilibrium where the asset price collapses and the decentralized matching market is disrupted due to a shortage of liquidity. In the bad equilibrium, the land price is low and the intratemporal debt, i.e., liquidity, dries up because of the shortage of collateral. Consequently, the decentralized matching market is shut down, since sellers choose not to participate in the market facing buyers' liquidity shortage. This market disruption lowers the aggregate productivity, while the low productivity justifies the low asset prices in turn. The asset price is lower and the debt/asset ratio, $b_t/(a_{t+1}K)$, is higher in the bad equilibrium. Therefore, a policy implication for financial crisis management is that if the government imposes the restriction that all impatient agents must set their debt burden, b_t, such that $b_t \leq a_{t+1}k_t - p_h q_h$, the crisis state is then eliminated and the economy may jump to the normal state. This result seems to support the effectiveness of debt reduction policy as financial crisis management. Thus our analysis in this paper may shed some light on the assessment of policies for financial crisis management that are explicitly concerned with asset prices and the aggregate amount of debt.

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A Appendix

In this Appendix, we confirm that our results in the previous section hold even if we introduce money into our model. As is the case for Carlstrom and Fuerst's (1997) model, the setting of our model with two different types of agents, patient and impatient, implies that impatient agents do not hold cash for payment in a steady-state equilibrium. In this Appendix, we consider a slightly stronger form of money than cash. We confirm that money does not matter for our results in a modified version of our model in which we introduce deposit money that can earn interest.

We assume that agents can hold intertemporal bank deposits that earn interest at the rate r_t^d, instead of bonds. Impatient agents can use the principal and interest from their deposits as a payment instrument in the decentralized market in addition to the bank notes that they borrow from banks.

A.1 Optimization of patient agents

Since patient agents hold bank deposit, d_t, instead of bonds, b_t, we get the Bellman equations for our monetary model by substituting d_t and r_t^d for b_t and r_t in the previous section. Note that d_t is measured in the units of the date-$(t+1)$ consumption good.

$$V^p(d_{t-1}) = \max\{-\kappa + \alpha(\mu_t)[-\gamma(q_t) + W^p(p_t q_t, d_{t-1}) - W^p(0, d_{t-1})], \quad 0\} + W^p(0, d_{t-1}), \tag{85}$$

and

$$W^p(l_t, d_{t-1}) = \max_{d_t} d_{t-1} - \frac{d_t}{1+r_t^d} + l_t + \beta V^p(d_t). \tag{86}$$

Similar arguments as those in the previous section imply

$$\beta(1+r_t^d) = 1. \tag{87}$$

A.2 Optimization of impatient agents

Since impatient agents can hold bank deposits, d'_t, as their assets, we need to include d'_t as the state variables. In this modified model, impatient agents borrow intertemporal loans, b_t, from banks instead of borrowing from patient agents directly. The interest rate for b_t is r_t, where $r_t \geq r_t^d$. The Bellman equation for the day market is

$$\begin{aligned} V(l_{t-1}, b_{t-1}, d'_{t-1}, k_{t-1}) =& \alpha(\mu_t)\mu_t \max_{y_t, q_t, l_t, d''_t} W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) \\ &+ (1-\alpha(\mu_t)\mu_t) W(0, l_{t-1}, l_{t-1} + d'_{t-1}, b_{t-1}, k_{t-1}), \end{aligned} \tag{88}$$

$$\text{subject to} \quad n'_t = d'_{t-1} + a_t k_{t-1} - p_t q_t, \tag{89}$$

$$n'_t \geq 0, \tag{90}$$

$$y_t = A k_{t-1}^{\theta} q_t^{1-\theta}, \tag{91}$$

where n'_t is the total of bank notes and deposits that the impatient agent brings into the night market. The Bellman equation for the night market is

$$\begin{aligned} W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{c'_t, l_t, b_t, d'_t, k_t} & U(c'_t) - c'_t + y_t - a_t\{k_t - k_{t-1}\} + \omega_t k_{t-1} + n'_t - \frac{d'_t}{1+r_t^d} - R_t l_t \\ &+ \frac{b_t}{1+r_t} - b_{t-1} + \beta' V(l_t, b_t, d'_t, k_t), \end{aligned} \tag{92}$$

subject to $b_t + R_t l_t \leq a_{t+1} k_t + d'_t$. FOCs and envelope conditions for b_t and d'_t imply

$$1 = (1+r_t)\xi_t + \beta'(1+r_t)(1 + \alpha(\mu_{t+1})\mu_{t+1} x_{t+1}), \tag{93}$$

$$1 \geq \beta'(1+r_t^d)\{1 + \alpha(\mu_{t+1})\mu_{t+1}[R_{t+1} + R_{t+1} x_{t+1} - 1]\}, \tag{94}$$

where $d'_t = 0$ if (94) holds with strong inequality. Therefore, equations (93) and (94) imply that $d'_t = 0$ if $R_{t+1} = 1$ and $r^d_t < r_t$, which, we will show below, is the case in the equilibrium.

A.3 Optimization of banks

Banks accept deposits d_t from patient agents and d'_t from impatient agents, and invest them into intertemporal lending b_t and the real cash reserve m_t under a technological constraint that total deposits[8] cannot exceed ϕm_t where ϕ ($\phi > 1$) is a technological parameter for production of payment services. The optimization problem of a bank in period t is written as follows:

$$\max_{l_t, b_t, d_t, d'_t} b_t + \frac{m_t}{1+\pi_t} - (d_t + d'_t) + (R_t - 1)l_t, \tag{95}$$

$$\text{subject to } \frac{b_t}{1+r_t} + m_t + l_t \leq \frac{d_t + d'_t}{1+r^d_t} + l_t, \tag{96}$$

$$\frac{d_t + d'_t}{1+r^d_t} \leq \phi m_t, \tag{97}$$

where π_t is the inflation rate. Note that l_t denotes the bank notes lent to impatient agents. The bank's problem can be rewritten as follows: Given r_t, r^d_t and R_t,

$$\max_{l_t, m_t} [(r_t - r^d_t)\phi + (1+\pi_t)^{-1} - (1+r_t)]m_t + (R_t - 1)l_t. \tag{98}$$

Since $l_t < \infty$ and $m_t < \infty$ in the equilibrium,

$$R_t = 1, \tag{99}$$

$$1 + r_t = \frac{\beta^{-1}\phi - (1+\pi_t)^{-1}}{\phi - 1}. \tag{100}$$

As long as $1 + \pi_t \geq \beta$, r_t exceeds $r^d_t = \beta^{-1} - 1$, and therefore, $d'_t = 0$ from (93) and (94). It is easily shown that qualitatively the same results as those in Section 2 hold for the

[8] Here we make a slightly problematic assumption that the intraperiod bank notes, l_t, that the bank issues in the day market are not constrained by cash holdings of the bank. Otherwise the constraint (97) would be $d_t + d'_t + l_t \leq \phi m_t$ and the intraperiod rate of interest would exceed one, that is, $R_t > 1$. In this case, the analysis would be complicated and our results would hold only for a certain range of parameter values. For simplicity, we assume in this paper that the constraint for the bank is (97).

steady-state equilibria of the model in this Appendix as long as $1 + \pi \geq \beta$, since $d_t' = 0$ in this case.

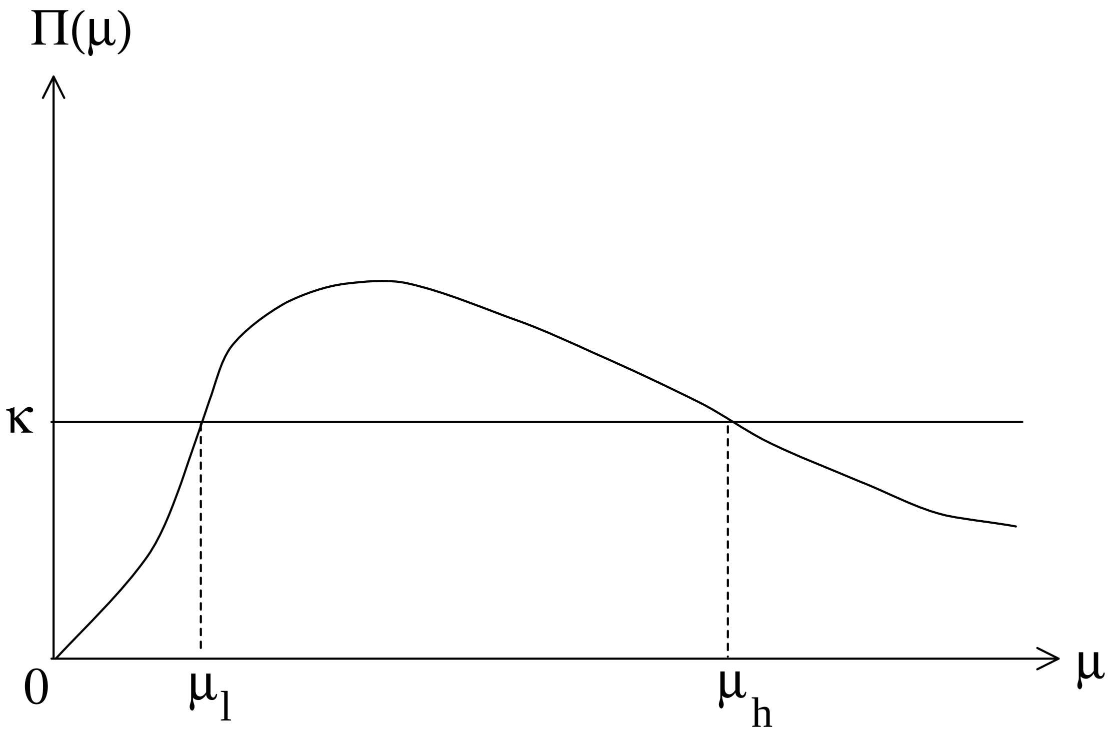

Figure 1: Determination of the measure of participating patient agents in Normal State