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Quantitative Significance of Collateral Constraints as an Amplification Mechanism*

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Abstract

Do large fluctuations arise from small shocks through financial frictions? In previous literature it is shown that a collateral constraint on intertemporal debt for consumption smoothing does not have a quantitatively significant effect on the response of output to unexpected shocks. We additionally focus on the collateral constraint on intratemporal debt for wage payments and examine the amplification of output. We find that output is significantly amplified in a standard functional form and parameter region. We also find that the region of the parameters for which the output is amplified is wider than that of previous literature.

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1 Introduction

Do large fluctuations arise from small shocks through financial frictions? In the previous literature, it is shown that when we focus on the collateral constraint on debt for consumption smoothing, financial frictions do not have a quantitatively significant effect on the response of output to unexpected shocks. Kiyotaki and Moore (1997) (hereafter KM) show that in the case of a collateral constraint on financing debt for consumption smoothing, a small shock to the total factor productivity (TFP) makes the output fluctuate substantially. However, Cordoba and Ripoll (2004) (hereafter CR) point out that the assumptions in KM are not standard: linear preferences of lender and linear production technology in land. And they show that, when we employ more standard specifications of preferences and technologies, KM's collateral constraint does not have a quantitatively significant effect on the response of output to unexpected shocks. Kocherlakota (2000) also points out similar results.

In this paper, we additionally focus on an alternative specification of the collateral constraint, that is, the collateral constraint on financing working capital (especially wage payments) and examine the amplification of the output. Borrowing for wage payments is limited by the value of collateralizable capital, since borrowers for wage payments cannot credibly commit to repay their debt. Under this assumption, when the TFP shock occurs, the increase in the price of collateral assets relaxes the collateral constraint on financing wage payments. Hence, constrained agents can hire more labor input and produce more. Therefore, the collateral constraint on financing wage payments has a mechanism that makes the response of output large. We find that the output is significantly amplified by small shocks even in the standard functional forms and the standard parameter region. We also find that the region of the parameters is wide where the output is amplified.

The following is a description of the related literature. In the models of KM and CR, collateral lending smooths consumption and investment intertemporally. This type of collateral constraint is used by Bernanke, Gertler, and Gilchrist (1999), Kocherlakota (2000), and Iacoviello (2005). Other papers (Carlstrom and Fuerst 1998; Jermann and Quadrini 2006; Mendoza 2006; Cheng and Song 2007; Kobayashi, Nakajima and Inaba 2007; Kobayashi and Nutahara 2007) focus on the collateral constraint on intra-period debt that is used as an instrument for factor payment for agents, and show that financial factors play a key role in generating economic fluctuations. We focus on the latter type of collateral constraint as financial frictions.

The paper is organized as follows. In Section 2, we provide a model with a collateral constraint on financing wage payments and examine the amplification of output due to the financial frictions. Section 3 reviews the benchmark model of CR and shows the reason why their model doesn't have the amplification effect. And we also consider which models are consistent with empirical evidence. The conclusion is given in Section 4.

2 Models with collateral constraint on financing wage payments

2.1 A model with CR's utility function

We employ a model with a collateral constraint on financing wage payments. The model is similar to that of CR except for the collateral constraint. We assume the economic environment as follows. Our model economy is a closed economy consisting of two types of agents (households) and banks, whose measures are normalized to one, respectively. Two types of continuum agents differ in their discount factor of preference, $\beta_1 > \beta_2$. Since type-1 agents are more patient than type-2, type-1 agents become lenders of the intertempral lending b_t for consumption smoothing, and type-2 agents become borrowers.[1] We assume that the labor market for each type of agent is separated from each other and that each agent cannot use his own labor force for his production, which means that a type i agent must hire labor input from the same type agent. Both types of agents produce output $y_{i,t}$ using a Cobb-Douglas production function, $y_{i,t} = A_t k_{i,t-1}^{\alpha_i} \ell_{i,t}^{\prime 1-\alpha_i}$, where $k_{i,t-1}$ denotes initial capital stock for type i agent at the beginning of time t, $\ell'_{i,t}$ denotes labor input from another agent in the same type, A_t denotes the productivity, and $0 < \alpha_i < 1$. Aggregate capital stock is constant over time. In addition, we assume that agents must pay for the inputs in advance of production. Following Kobayashi, Nakajima, and Inaba (2006), we assume that a bank can issue bank notes that can be circulated in the economy as payment instruments, and that agents need to borrow bank notes for payments for the inputs. Let $n_{i,t}$ be the amount of bank notes that type-i agents borrow for wage payments. Then, given $n_{i,t}$, the agents choose $\ell'_{i,t}$ under the constraint:

$$w_{i,t}\ell'_{i,t} \leq n_{i,t}, \tag{1}$$

where $w_{i,t}$ denotes the real wage for type-i agent's labor force. Borrowing and lending for wage payments are intra-period: the agents are supposed to repay $R_t n_{i,t}$ after production, where R_t is the gross rate of bank loans.

Finally, we assume that both types of agents are subject to the collateral constraint on financing debt for wage payments. Since agents cannot fully commit themselves to repay the debt for wage payments, they can escape with the output without any other penalty than losing their collateralizable capital. As a result, agents have to provide their own collateralized capital as collateral to the bank. The details for the collateralized capital are as follows. We assume that the intertemporal debt $(1 + r_{t-1})b_{t-1}$ is a senior debt and the intratemporal debt $R_t n_{i,t}$ a junior debt. Since type-1 agents are lenders and type-2 agents are borrowers for the intertemporal debt, the collateralizable capital for agents differs. For type-1

[1] Note that b_t denotes lending and is always positive for the convenience of simulation.

agents, the collateralizable capital consists of the value of capital stock, $q_t k_{1,t-1}$, and gross interest income for intertemporal lending, $(1+r_{t-1})b_{t-1}$, at the beginning of period t. Therefore, the collateral constraint on borrowing the bank notes for type-1 agents is defined as

$$R_t n_{1,t} \leq \phi[q_t k_{1,t-1} + (1 + r_{t-1})b_{t-1}]. \tag{2}$$

where ϕ is the ratio of respective assets that can be put up as collateral. For type-2 agents, the collateralizable capital consists of the value of capital stock, $q_t k_{2,t-1}$, minus principal and interest repayments for inter-period lending, $(1 + r_{t-1})b_{t-1}$, at the beginning of period t. Therefore, the collateral constraint on borrowing the bank notes for type-2 agents is defined as

$$R_t n_{2,t} \leq \phi\left[q_t k_{2,t-1} - (1 + r_{t-1})b_{t-1}\right]. \tag{3}$$

The bank's problem is to maximize the return on the loan, $(R_t - 1)(n_{1,t} + n_{2,t})$. Since the bank faces no risk of default if the intra-period loan satisfies (2) and (3), competition among banks implies that the return on the loan should be zero $(R_t - 1 = 0)$ in equilibrium. Therefore, in equilibrium the banks become indifferent to the amount of $n_{1,t} + n_{2,t}$ and work as passive liquidity suppliers to the agents. We can neglect the banks' decision-making since it has no effect on the equilibrium dynamics of this economy. Conditions (1) and (2) together imply the following collateral constraint on wage payments for type-1 agents:

$$w_{1,t}\ell'_{1,t} \leq \phi\left[q_t k_{1,t-1} + (1 + r_{t-1})b_{t-1}\right]. \tag{4}$$

Conditions (1) and (3) together imply the following collateral constraint on wage payments for type-2 agents:

$$w_{2,t}\ell'_{2,t} \leq \phi\{q_t k_{2,t-1} - (1 + r_{t-1})b_{t-1}\}. \tag{5}$$

Here, we assume the parameter region where the collateral constraint does not bind for type-1 agents.

At the end of period t, after production and repayment for $R_t n_{i,t}$, each agent repays $(1+r_{t-1})b_{t-1}$, and determines consumption, $c_{i,t}$, capital stock, $k_{i,t}$, and intertemporal debt, $b_{i,t}$, subject to the flow budget constraint. The budget constraint for type-1 agents is described as

$$\begin{aligned} c_{1,t} + q_t(k_{1,t} - k_{1,t-1}) + b_t + R_t n_{1,t} \\ = A_t k_{1,t-1}^{\alpha_1} \ell_{1,t}'^{1-\alpha_1} + (1+r_{t-1})b_{t-1} + n_{1,t} + w_{1,t}\ell_{1,t} - w_{1,t}\ell_{1,t}', \end{aligned} \tag{6}$$

where $c_{i,t}$ denotes consumption of type i, $\ell_{i,t}$ denotes the supply of labor in the type-i labor market, $\ell_{i,t}'$ denotes the labor input to produce, q_t denotes the price of capital, and r_t is the rate of return on lending. Since $R_t = 1$ because of the competition among banks, the reduced form of the budget constraint for type-1 agents is

$$c_{1,t} + q_t(k_{1,t} - k_{1,t-1}) + b_t = A_t k_{1,t-1}^{\alpha_1} \ell_{1,t}'^{1-\alpha_1} + (1+r_{t-1})b_{t-1} + w_{1,t}\ell_{1,t} - w_{1,t}\ell_{1,t}'. \tag{7}$$

In a similar manner, the budget constraint for type-2 agents is described as

$$\begin{aligned} c_{2,t} + q_t(k_{2,t} - k_{2,t-1}) - b_t + R_t n_{2,t} \\ = A_t k_{2,t-1}^{\alpha_2} \ell_{2,t}'^{1-\alpha_2} - (1+r_{t-1})b_{t-1} + n_{2,t} + w_{2,t}\ell_{2,t} - w_{2,t}\ell_{2,t}', \end{aligned} \tag{8}$$

where b_t denotes the borrowing from type-1 agents by type-2 agents. Since $R_t = 1$, the reduced form of the budget constraint for type-2 agents is

$$c_{2,t} + q_t(k_{2,t} - k_{2,t-1}) - b_t = A_t k_{2,t-1}^{\alpha_2} \ell_{2,t}'^{1-\alpha_2} - (1+r_{t-1})b_{t-1} + w_{2,t}\ell_{2,t} - w_{2,t}\ell_{2,t}', \tag{9}$$

where b_t denotes borrowing by type-2 agents.

Type-i agents maximize their lifetime utility described as

$$E_0 \sum_{t=0}^{\infty} \beta_i^t \frac{\left(c_{i,t} - \frac{1}{\gamma_i}\ell_{i,t}^{\gamma_i}\right)^{1-\sigma_i}}{1-\sigma_i}, \qquad \text{for } i = 1,\ 2,$$

Sequence	Event
1	Type-i Agents borrow $n_{i,t}$ for the wage payments
2	Type-i Agents employ labor $\ell'_{i,t}$ by paying $w_t\ell'_{i,t}$
3	Type-i Agents produce $A_t k^{\alpha}_{i,t-1}\ell'^{1-\alpha}_{i,t}$
4	Type-i Agents repay $R_t n_{i,t}$ to banks
5	Type-i Agents clear $(1+r_{t-1})b_t$ and choose $c_{i,t}$, $k_{i,t}$, and b_t

Table 1: The time table for events in period t

subject to the collateral constraint and budget constraint: and (4) and (7) for type-1 agents, and (5) and (9) for type-2 agents. The time table for events in period t is summarized in Table 1.

The market clearing conditions are described as follows.

$$\text{(Goods) } c_{1,t} + c_{2,t} = y_{1,t} + y_{2,t}, \tag{10}$$

$$\text{(Labor) } \ell'_{i,t} = \ell_{i,t}, \quad \text{(for } i = 1, 2), \tag{11}$$

$$\text{(Capital) } k_{1,t} + k_{2,t} = 1. \tag{12}$$

Finally, the exogenous process of productivity is common for both types of agents:

$$\log A_{t+1} = (1-\rho_0)\log \overline{A} + \rho \log A_t + \epsilon_{t+1}, \tag{13}$$

where ϵ is a i.i.d. shock, and $\overline{A} = 1$.

The competitive equilibrium is defined as the allocation $\{c_{i,t},\ \ell_{i,t},\ k_{i,t},\ b_t\}_{t=0}^{\infty}$ for $i = 1, 2$ and prices $\{q_t,\ r_t,\ w_t\}_{t=0}^{\infty}$ that satisfy the following two statements. (i) Both types of agents maximize their lifetime utility subject to (4) and (7) for type-1 agents, and to (5) and (9) for type-2 agents. (ii) The market clearing conditions hold.

The equilibrium system is as follows:

$$\left(c_{1,t} - \frac{1}{\gamma_1}\ell_{1,t}^{\gamma_1}\right)^{-\sigma_1} = \lambda_{1,t}, \tag{14}$$

$$\ell_{1,t}^{\gamma_1 - 1} = w_{1,t}, \tag{15}$$

$$(1+\mu_{1,t})w_{1,t} = (1-\alpha_1)\frac{y_{1,t}}{\ell'_{1,t}}, \tag{16}$$

$$\lambda_{1,t}q_t = \beta_1 E_t\left\{\lambda_{1,t+1}\left[(1+\phi\mu_{1,t+1})q_{t+1} + \alpha_1\frac{y_{1,t+1}}{k_{1,t}}\right]\right\}, \tag{17}$$

$$\lambda_{1,t} = \beta_1 E_t\left[\lambda_{1,t+1}(1+\phi\mu_{1,t+1})(1+r_t)\right], \tag{18}$$

$$\left(c_{2,t} - \frac{1}{\gamma_2}\ell_{2,t}^{\gamma_2}\right)^{-\sigma_2} = \lambda_{2,t}, \tag{19}$$

$$\ell_{2,t}^{\gamma_2 - 1} = w_{2,t}, \tag{20}$$

$$(1+\mu_{2,t})w_{2,t} = (1-\alpha_2)\frac{y_{2,t}}{\ell'_{2,t}}, \tag{21}$$

$$\lambda_{2,t}q_t = \beta_2 E_t\left\{\lambda_{2,t+1}\left[(1+\phi\mu_{2,t+1})q_{t+1} + \alpha_2\frac{y_{t+1}}{k_{2,t}}\right]\right\}, \tag{22}$$

$$\lambda_{2,t} = \beta_2 E_t\left[\lambda_{2,t+1}(1+\phi\mu_{2,t+1})(1+r_t)\right], \tag{23}$$

$$w_{1,t}\ell'_{1,t} \leq \phi[q_t k_{1,t-1} + (1+r_{t-1})b_{t-1}], \tag{24}$$

$$w_{2,t}\ell'_{2,t} \leq \phi\left[q_t k_{2,t-1} - (1+r_{t-1})b_{t-1}\right], \tag{25}$$

$$y_{1,t} = A_t k_{1,t-1}^{\alpha_1}\ell_{1,t}'^{1-\alpha_1}, \tag{26}$$

$$y_{2,t} = A_t k_{2,t-1}^{\alpha_2}\ell_{2,t}'^{1-\alpha_2}, \tag{27}$$

$$\ell'_{i,t} = \ell_{i,t}, \quad (\text{for } i = 1,2), \tag{28}$$

$$c_{1,t} + c_{2,t} = y_{1,t} + y_{2,t}, \tag{29}$$

$$c_{2,t} + q_t(k_{2,t} - k_{2,t-1}) - b_t + w_{2,t}\ell'_{2,t} = y_{2,t} - (1+r_{t-1})b_{t-1} + w_{2,t}\ell_{2,t}, \tag{30}$$

$$\log A_{t+1} = \rho_0 + \rho\log A_t + \epsilon_{t+1}. \tag{31}$$

2.1.1 Result of amplification

We log-linearize the model around the deterministic steady state. The simulation setting is as follows.

- At time $t = -1$, the economy is in the steady state.
- At time $t = 0$, one percent of the TFP shock temporally occurs in both types of agents.
- For a different pair of parameters, the share of capital $\alpha \in (0,\ 1)$ and the inverse of the intertemporal substitution elasticity of consumption $\sigma \in (0,\ 3)$, we measure the percentage change in output from the steady state level with respect to the TFP shock at time $t = 0$, which means the elasticity of output to the TFP shock.

The parameter settings are as follows. The discount factor for type-1 agents, β, is set to be 0.9. The discount factor for type-2 agents, β', is set to be $0.9 \times \beta$. σ_1 and σ_2 are set to be 1. The ratio for the collateral value, ϕ, is set to be 1. $\gamma_1 = \gamma_2$ is set so that the steady state value of aggregate labor is 1. Finally, the exogenous process of the productivity has no persistence: $\rho = 0$.

Since the only source of output variation in CR's benchmark model is capital allocation, CR define the amplification of output as follows: the elasticity of output to TFP is greater than one in the next period when a TFP shock occurs. However, in the case of flexible labor choice, since the labor input can change at the same time when the TFP shock occurs, CR's definition of amplification is not suitable. We employ an alternative definition of the amplification: the elasticity of output to TFP is greater than two at the time when a TFP shock occurs.[2] In order to maintain comparability between the model, output is considered to be amplified when the elasticity of output is greater than one excluding the effect of a 1% TFP shock.

Figure 1 illustrates the magnitude of output amplification in our model for different pairs $(\alpha,\ \sigma)$. We shaded the parameter region where output is amplified. As

[2] In Appendix A, we apply our definition of amplification to CR's model with flexible labor choice and show that it has a quantitatively significant amplification effect.

Figure 1 shows, the region of amplified output is wide.[3] In addition, the elasticity of output to the TFP shock is greater than two for standard parameter values, for example $\alpha = 0.3$ and $\sigma = 1$.

There are two parts to the intuitive interpretation of the results. The first one is based on the flexible labor choice. In this model, the economic agents can choose the labor supply and labor demand optimally. When a TFP shock occurs, the demand for the labor input shifts to the right given the real wage rate. Hence, the labor input and the real wage rate in equilibrium increase. Therefore, the output increases. The second one is based on a collateral constraint to finance the payment for labor input. When the TFP shock occurs, the increase in the price of capital relaxes the collateral constraint on financing wage payments. Hence, constrained agents can hire more labor input, produce more output, and the response of output can be amplified by the financial frictions. Therefore, the interaction between financial market inefficiency and labor market inefficiency is crucial to our model.

2.2 A model with a CRRA utility function

In order to check the robustness for the specifications of the utility function, we employ a constant relative risk aversion (CRRA) utility function: $\frac{\left[c_{i,t}^{1-\gamma_i}(1-\ell_{i,t})^{\gamma_i}\right]^{1-\sigma_i}}{1-\sigma_i}$, for $i = 1, 2$. Since the balanced growth path is defined, this utility function might be more standard than the previous one. For the parameter settings, $\gamma_1 = \gamma_2$ is set so that the steady state value of aggregate labor is $\frac{1}{3}$. The other settings for the model and simulation procedure are the same as above. As shown in Figure 2, the region of parameters where the elasticity is greater than two is also wide. Besides, for standard parameters, for example $\alpha = 0.3$ and $\sigma = 1$, the elasticity is greater than two.

[3] It is not necessary that the amplified regions of parameters overlap in both cases because the features of the collateral constraint are different from each other.

3 Discussion

3.1 Why CR's model doesn't have an amplification effect.

We consider why the benchmark model of CR doesn't have an amplification effect. The reason is because the maximum value of aggregate output in their model is not far from the value of aggregate output in the steady state where the collateral constraint is binding. We demonstrate this below. The simulation setting of CR is as follows.

- At time $t = -1$, the economy is in the steady state at which the collateral is binding.
- At time $t = 0$, one percent of TFP shock temporally occurs in both types of agents.
- Measure the percentage change in output from the steady state level with respect to a 1% TFP shock at time $t = 1$, in other words the elasticity of output to the TFP shock.

The aggregate output of CR is defined as

$$Y_t = A_{1,t} k_{1,t-1}^{\alpha_1} + A_{2,t} k_{2,t-1}^{\alpha_2}, \tag{32}$$

where $k_{1,t-1} + k_{2,t-1} = 1$. Note that in CR's benchmark model labor supply is fixed at one. Although the capital input does not change at the time when the TFP shock occurs, the allocation of capital stock changes in the next period. We assume that $A_1 = A_2$ and $\alpha_1 = \alpha_2$. The TFP shock occurs in both types of agents at time $t = 0$ as an impulse. Then, the level of TFP goes back to the stationary level, $A = 1$, at $t = 1$. Therefore, the source of change in aggregate output is only capital allocation. In order to make clear the effect of capital allocation in this model, we assume that agents can choose an allocation of capital stock at $t = 1$ so as to maximize the elasticity of output to a TFP shock. Then, the maximum elasticity of aggregate output to the 1% shock to TFP evaluated in the steady state for $\alpha \in (0,1)$ is defined as follows:

$$\max_{\Delta k_2} \left\{ \frac{[1 - m_2(k_2^* + \Delta k_2)]^\alpha + (k_2^* + \Delta k_2)^\alpha - (y_1^* + y_2^*)}{y_1^* + y_2^*} \times 100 \right\}. \tag{33}$$

Then,

$$\arg\max_{\Delta k_2} \frac{Y_{(t=1)} - Y^*}{Y^*} \times 100 = \left(\frac{1}{2} - k_2^* \right). \tag{34}$$

The superscript $*$ stands for the collateral-constrained steady state value. Since this elasticity can be interpreted as the percentage discrepancy of aggregate output from collateral-constrained steady state to the first best steady state, we examine the model's potential elasticity of output to the TFP shock by using this value. We plot the maximum elasticity with respect to α in Figure (1). The other parameter settings are as follows: $\beta = 0.9$ and $\beta' = 0.9 \times \beta$. As we show in Figure 3, the elasticity of output is greater than one for only high values of α, which is a similar result to that of CR. This finding implies that the benchmark model of CR does not have a significant amplification mechanism in standard parameter values, even when the output increases as it can.

3.2 Which specification is consistent with empirical evidence?

We have described two specifications of collateral constraints as financial frictions: the collateral constraint on financing debt for consumption smoothing and the collateral constraint on financing wage payments. Here, we consider which specification is supported by empirical evidence. According to the results of business cycle accounting of Chari, Kehoe, and McGrattan (2007) and Kobayashi and Inaba (2006), the labor wedge becomes large for the U.S. in the 1930 and for Japan in the 1990s and plays an important role in the output fluctuations. The labor wedge is defined as the marginal rate of substitution divided by marginal product of labor in a standard real business cycle model. Models of CR do not affect the labor wedge, because there are no frictions on the labor market. On the other hand, in our models, the collateral constraint on financing wage payments changes the marginal product condition on labor as a friction of the labor market and affects the labor wedge. Therefore, the simulation results of our models might be consistent with the findings of BCA.

4 Conclusion

We examined whether large fluctuations arise from small shocks through financial frictions, focusing on the collateral constraint on financing wage payments. We found that small shocks amplify the output significantly in a standard parameter region. These findings support the viewpoint that credit market frictions have important effects in business cycles.

Appendix A Model with labor input in CR

Here we introduce the labor choice into the benchmark model of CR, and investigate the amplification effects on the output using our definition of amplification. While the only source of output variation in the benchmark model is capital allocation, the flexible labor choice can be another source. CR also consider the same model as that of this appendix, but show that the magnitude of amplification is small according to their definition of amplification. However, since the labor input can change at the same time as the TFP shock occurs, CR's definition of amplification is not suitable for this model. We will show that when we employ our definition of amplification the output can be amplified by a small shock to the TFP.

The economic environment is the same as in CR with labor choice. There are two types of continuum agents who differ in their rate of preference. Population size of each type is $m_i > 0$, $i = 1, 2$. Agents may also differ in other dimensions such as the degree of risk aversion or production technology. There are two types of goods: a durable asset (capital, k), and a non-durable commodity (consumption goods, c). Both types of agents produce using a concave technology, $f(k_{i,t-1}, \ell_{i,t})$. Borrowers cannot commit to repay their loans, b_t. They can escape with the production with no other penalty than loss of their capital. As a result, loans need to be secured by the value of the capital. Aggregate capital is constant over time. $K_{t-1} = k_{1,t-1} + k_{2,t-1} = \overline{K}$. Finally, the labor market is separated.

Type-1 agents maximize their lifetime utility:

$$\max_{c_{1,t},\ell_{1,t},k_{1,t},b_t} E_0 \sum_{t=0}^{\infty} \beta_1^t \frac{\left(c_{1,t} - \frac{1}{\gamma_1}\ell_{1,t}^{\gamma}\right)^{1-\sigma_1}}{1-\sigma_1},$$

$$\text{s.t. } c_{1,t} + q_t(k_{1,t} - k_{1,t-1}) + b_{t-1} = f_1(k_{1,t-1}) + (1 + r_{t-1})b_{t-1},$$

$$f_1(k_{1,t-1}, \ell_{1,t}) = k_{1,t-1}^{\alpha_1}\ell_{1,t}^{1-\alpha_1},$$

$$b_t \leq E_t(\phi q_{t+1} k_{1,t}),$$

where $1 > \beta_1 > \beta_2 > 0$, c_1 is the consumption of type-1 agents, q is the price of capital, b_t are debt payments (including interest), and r_t is the rate of return.

Type-2 agents maximize their lifetime utility:

$$\max_{c_{2,t},\ell_{2,t},k_{2,t},b_t} E_0 \sum_{t=0}^{\infty} \beta_2^t \frac{\left(c_{2,t} - \frac{1}{\gamma_2}\ell_{2,t}^{\gamma}\right)^{1-\sigma_2}}{1-\sigma_2},$$

$$\text{s.t. } c_{2,t} + q_t(k_{2,t} - k_{2,t-1}) + b_{t-1} = f_2(k_{2,t-1}) + (1 + r_{t-1})b_{t-1},$$

$$f_1(k_{2,t-1}, \ell_{1,t}) = k_{2,t-1}^{\alpha_2} \ell_{2,t}^{1-\alpha_2},$$

$$b_t \leq E_t(\phi q_{t+1} k_{2,t}),$$

where $1 > \beta_1 > \beta_2 > 0$, c_2 is the consumption of type-2 agents, q is the price of capital, b_t are debt payments (including interest), and r_t is the rate of return.

Figure A.1 illustrates the magnitude of output amplification in our model for different pairs $(\alpha,\ \sigma)$. For the parameter settings, $\gamma_1 = \gamma_2$ is set so that the steady state value of aggregate labor is 1. The parameter settings are as follows. The discount factor for type-1 agents, β, is set to 0.9. The discount factor for type-2 agents, β', is set to $0.9 \times \beta$. σ_1 and σ_2 are set to 1. Finally, the exogenous process of the productivity has no persistence: $\rho = 0$. The other parameters are the same as above. We have shaded the parameter region where the elasticity of output to the TFP shock is greater than two.

When we employ our definition of amplification, the output can be significantly amplified by the small shock to the TFP even in CR's model with the flexible labor choice. However, the parameter region where the output is significantly amplified is not wide and does not include the usual set of values, $(\sigma,\ \alpha) = (1,\ 0.3)$.

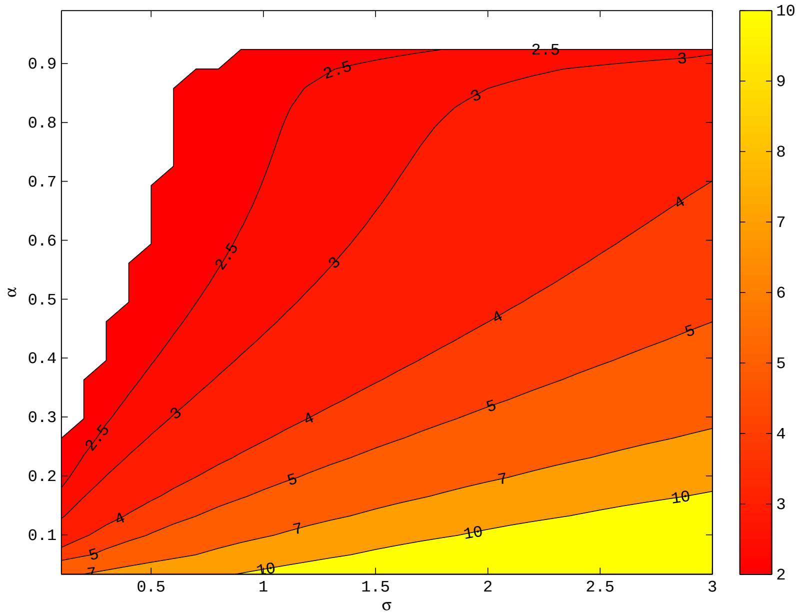

Figure 1: Region where the elasticity to the TFP shock is greater than 2.

Notes: The utility function is the same as that of CR.

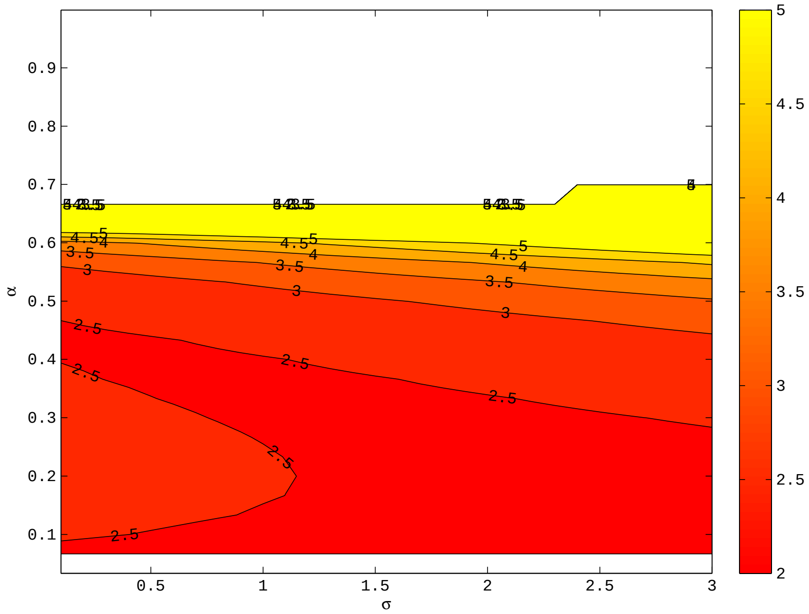

Figure 2: Region where the elasticity to the TFP shock is greater than 2.

Notes: The utility function is a CRRA type.

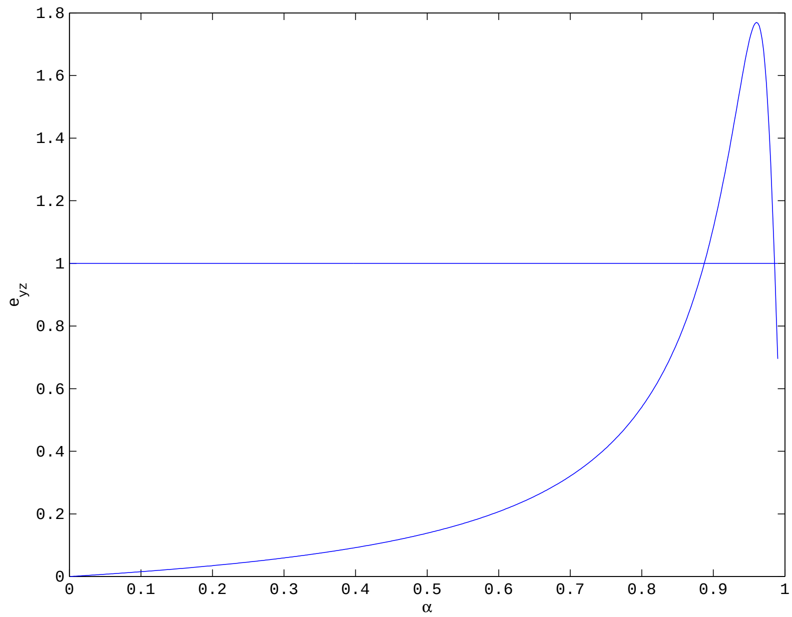

Figure 3: Elasticity of the output response to a TFP shock

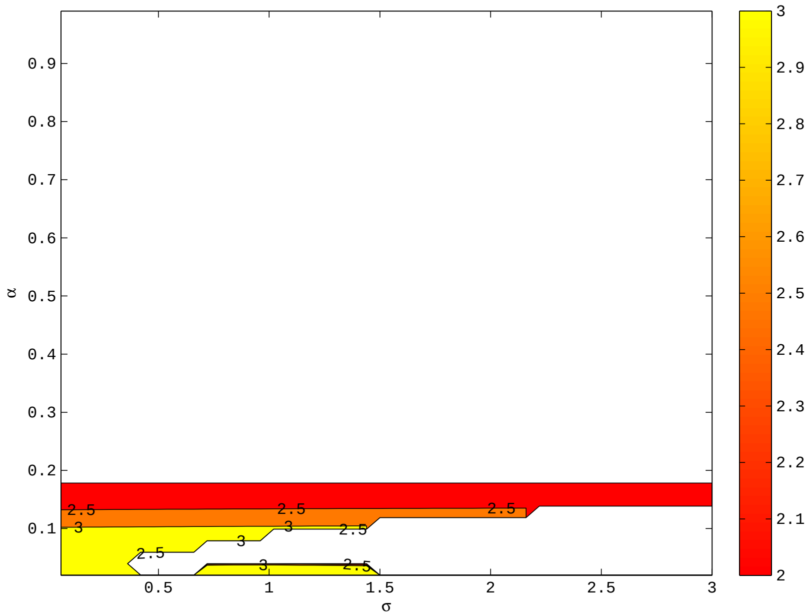

Figure A.1: Region where the elasticity to a TFP shock is greater than 2 in CR's model with labor.

Notes: The model is the same as that of CR.

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