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# Optimal Transmission Capacity under Nodal Pricing and Incentive Regulation for Transco\*

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## Abstract

This paper examines regulatory incentive mechanisms for efficient investment in the transmission network, taking into account both technological externalities among transmission lines and information asymmetry between the regulator and the transmission company (Transco). First, by adding extra constraints associated with the power flow, we develop an extended price cap mechanism that can internalize technological externalities among transmission lines. We show that this new mechanism induces the Transco to choose the optimal transmission capacity under its budget constraint. An extended form of the Vogelsang and Finsinger (V-F) mechanism is also introduced. Next, we examine the surplus-based scheme with government transfers. We provide a formal analysis of the incremental surplus subsidy (ISS) scheme specifically for the Transco, demonstrating that it induces the Transco to choose the optimal transmission capacity without the budget constraint.

Keywords: Transco, transmission capacity, nodal pricing, budget constraint, incentive regulation

JEL classification numbers: L51, L94

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## 1. Introduction

The competitive electricity market has developed at two ends: the generation end and the retail supply end. In contrast, the power transmission sector remains a monopoly business, and therefore subject to regulation. As is well known, there are significant economies of scale in network expansion, and hence the transmission sector has a natural tendency toward monopoly.

It is generally accepted that transmission regulation should seek to achieve the following primary principles:

- (i) Efficient utilization of the network in the short run.
- (ii) Efficient investment in the network in the long run.
- (iii) Cost recovery of the network assets.
- (iv) Provision of incentives for efficient investment.

It should be noted that, due to the complex nature of the power system, the regulation of the transmission network is a challenging task. For example, transmission regulation should take into account technological externalities among transmission lines, governed by physical laws (e.g., Kirchhoff's laws).

Regarding principle (i), there is a broad literature on transmission pricing that achieves optimal capacity utilization. Bohn et al. (1984) and Schweppe et al. (1988) introduce the concept of spot pricing, or nodal pricing. Nodal pricing is based on the optimal dispatch under the transmission capacity constraints, which is the application of marginal cost pricing. It can efficiently manage transmission congestion, and therefore achieve allocative efficiency in the short run. However, while nodal pricing aims to allocate resources efficiently in the short term, it needs some additional mechanism to induce optimal expansion of the network in the long term. Moreover, the total cost of transmission facilities must be recovered in the presence of substantial economies of scale.

It is a challenging task to fulfill principles (ii) to (iv), along with principle (i). The framework has been extended in two directions: the merchant transmission approach and the incentive regulation approach. For example, Rosellón (2003) discusses the two approaches in detail.

The merchant transmission approach, or the market-based transmission approach, relies on decentralized property-rights-based mechanisms to encourage transmission investment. This approach requires separation of transmission ownership and operation, and creates transmission rights for merchant investors based on the increase of the network capacity. Hogan (1992) and Bushnell and Stoft (1996, 1997) show that free entry by merchant investors can lead to efficient transmission investment that is profitable if a number of assumptions are met.<sup>1</sup> The main assumptions include the following: no economies of scale, the presence of well-defined property rights, the presence of a full set of futures markets, and no market power in the wholesale markets. However, the attractive properties of the merchant transmission approach will be seriously undermined if more realistic characterizations of the transmission network are considered. For example, Joskow and Tirole (2004) show how the efficiency of market-based transmission investment breaks down when some assumption such as no economies of scale is relaxed.

By contrast, the incentive regulation approach, or the regulated Transco approach, relies on regulatory incentive mechanisms that induce the transmission company (Transco) to expand the network efficiently. The Transco is generally defined as a regulated monopoly which is responsible for building, owning and operating transmission facilities. Although this approach is considered as an alternative or a complement to the merchant transmission approach, a limited number of such incentive schemes specifically for transmission regulation have been explored

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<sup>1</sup> While Hogan (1992) introduces financial transmission rights, Chao and Peck (1996) derive physical transmission rights.

in the literature.

One such scheme is a regulatory mechanism that confronts the Transco with some measure of the social gain associated with its activity, by using government subsidies (transfers). Gans and King (1999) suggest implementing the incremental surplus subsidy (ISS) scheme, proposed by Sappington and Sibley (1988), to induce efficient transmission investment. They simply suggest subsidizing the firm based on the increment of the social surplus, but do not formally describe the mechanism. Similarly, Joskow and Tirole (2002) provide insights on such a surplus-based scheme without a formal model. Léautier (2000) proposes a regulatory mechanism that is related to the surplus-based scheme. He begins by deriving the optimal transmission capacity under nodal pricing, without considering the Transco's budget constraint. In other words, he explicitly shows the first-best investment level, taking into account technological externalities among transmission lines. He next proposes a mechanism that attempts to confront the Transco with the social cost of congestion along with the expansion cost, which induces the optimal transmission capacity. However, the result depends on the applicability of the incentive-pricing dichotomy introduced by Laffont and Tirole (1993).<sup>2</sup> Moreover, the results of these mechanisms depend on whether the regulator is allowed to make transfers to the Transco.

On the other hand, considering the case without government subsidies, Vogelsang (2001) introduces a two-part price cap mechanism that attempts to induce the Transco to raise enough revenue for transmission investment, and at the same time, receive correct signals for efficient network expansion. He considers that the Transco would charge a two-part tariff for transmission services, and choose the fixed fee and the variable fee subject to the overall cap. The Transco then trades off congestion against capacity expansion in such a way that it becomes profitable to expand if the reduction in the congestion cost is greater than the cost of investment. Under several assumptions, Vogelsang argues that the transmission capacity will converge to the optimal level, while the variable fee equals the marginal congestion cost. However, since Vogelsang's mechanism does not take account of technological externalities among transmission lines, it would be difficult to achieve optimal expansion of the network. Therefore, it remains an unsettled question how to design appropriate price cap mechanisms in the presence of technological externalities. Moreover, if transmission customers are heterogeneous, the efficiency properties of this mechanism based on a two-part tariff could break down. Since the capital costs of transmission facilities are huge, the fixed fee sufficient to cover the costs would make a non-negligible number of customers drop out of the market, which causes allocative inefficiency.

The current paper develops dynamic mechanisms that are closely related to the Vogelsang's (2001) proposal. However, we take another route by considering linear tariffs for transmission services. Our discussion is based on nodal pricing, which seems to have become the efficiency standard for short-run energy and transmission pricing in the U.S. Under nodal pricing, the Transco can earn the congestion rent from congested transmission lines. In the long run, the congestion rent will change on average, in accordance with the increase or decrease in the transmission capacity. Therefore, by adjusting the transmission capacity appropriately, a linear congestion charge can be set in such a way that the congestion rent would recover the total cost of transmission facilities. Based on this simple principle, we start by deriving the optimal transmission capacity under the Transco's budget constraint. To put it another way, we show the second-best investment level under nodal pricing, by imposing a revenue constraint on the problem considered by Léautier (2000). Next, we develop a regulatory incentive mechanism for the Transco on the basis of a standard price cap mechanism. If there were no technological externalities, a typical price cap mechanism could be directly applied in a simple form of a constraint on the transmission capacity: the Transco might be allowed to choose the current

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<sup>2</sup> If the dichotomy property applies, the choice of the optimal transmission capacity can be separated from the incentive for cost reduction. However, the dichotomy property holds only for specific cost functions.

period's capacity and hence the congestion prices as long as a Laspeyres congestion price index would not be greater than a price cap. However, technological externalities among transmission lines must be taken into account appropriately in order to induce efficient expansion of the network. Therefore, we develop an extended form of the price cap mechanism by adding extra constraints associated with the power flow, so that technological externalities can be internalized. We show that the extended price cap mechanism can induce the Transco to choose the optimal transmission capacity under its budget constraint. We also introduce an extended form of the V-F mechanism proposed by Vogelsang and Finsinger (1979).

This paper also examines the surplus-based scheme, considering that the regulator can directly compensate the Transco for the deficit. We provide a formal analysis of the ISS scheme specifically for the Transco, and explicitly demonstrate that it induces the Transco to choose the optimal transmission capacity without its budget constraint, internalizing technological externalities among transmission lines.

The rest of the paper is organized as follows. In section 2, we present a model of a competitive power market, and characterize nodal pricing. In section 3, we derive the optimal transmission capacity under nodal pricing, with and without considering the Transco's budget constraint. In section 4, we propose extended forms of the price cap mechanism and the V-F mechanism. We show that these mechanisms can induce the Transco to choose the optimal transmission capacity under its budget constraint. In section 5, we provide a formal analysis of the ISS scheme specifically for the Transco, demonstrating that it can induce the Transco to choose the optimal transmission capacity without the budget constraint. Section 6 concludes.

## 2. The Model

### 2.1 Power Market

We consider an electric power network with  $N$  nodes and  $L$  transmission lines. The transmission capacity of each line is denoted by the vector  $\mathbf{k} \equiv (k^1, \dots, k^L)'$ .<sup>3</sup>

Consumers and generators make transactions in a competitive power market. Generators inject power at nodes, while consumers (or "loads") withdraw power at nodes. We consider an independent transmission company which owns and operates the transmission grid. In addition, this company maintains and expands the transmission network, and collects the revenues through charges levied on users. This sort of transmission company is often called a Transco. The Transco is a regulated, profit-making company, completely independent of all other market participants.

$q^{n,d}$  is the power demand at node  $n$  for  $n = 1, \dots, N$ .  $p^{n,d} = P^{n,d}(q^{n,d})$  and  $B^n(q^{n,d})$  are the inverse demand function and the gross benefit function at node  $n$ , respectively. We assume that  $B^n(q^{n,d})$  is twice continuously differentiable and a non-decreasing concave function. Note that  $\partial B^n(q^{n,d}) / \partial q^{n,d} = P^{n,d}(q^{n,d})$ . The consumers' surplus at node  $n$  can be expressed as  $CS^n(q^{n,d}) \equiv B^n(q^{n,d}) - P^{n,d}(q^{n,d})q^{n,d}$ . Furthermore, the aggregated consumers' surplus of all nodes can be written as  $CS(\mathbf{q}^d) \equiv \sum_{n=1}^N CS^n(q^{n,d})$ , where  $\mathbf{q}^d \equiv (q^{1,d}, \dots, q^{N,d})'$ .

$q^{n,s}$  is the power generation at node  $n$ .  $p^{n,s} = P^{n,s}(q^{n,s})$  and  $G^n(q^{n,s})$  are the marginal and total cost of generation at node  $n$ , respectively. We assume that  $G^n(q^{n,s})$  is

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<sup>3</sup>  $\mathbf{a}'$  denotes the transpose of matrix (or vector)  $\mathbf{a}$ .

twice continuously differentiable and a non-decreasing convex function. Clearly,  $\partial G^n(q^{n,s})/\partial q^{n,s} = P^{n,s}(q^{n,s})$  holds. The generators' profit at node  $n$  can be expressed as  $\Pi^n(q^{n,s}) \equiv P^{n,s}(q^{n,s})q^{n,s} - G^n(q^{n,s})$ . Moreover, the aggregated generators' profit of all nodes can be written as  $\Pi(\mathbf{q}^s) \equiv \sum_{n=1}^N \Pi^n(q^{n,s})$ , where  $\mathbf{q}^s \equiv (q^{1,s}, \dots, q^{N,s})'$ .

The Transco buys power from generators at nodes, transmits it through the grid, and sells it to consumers at nodes. Therefore, the difference between the revenues from consumers and the payments to generators is the Transco's surplus at each node, i.e.,  $MS^n(q^{n,d}, q^{n,s}) \equiv P^{n,d}(q^{n,d})q^{n,d} - P^{n,s}(q^{n,s})q^{n,s}$ . Furthermore, the aggregated Transco's surplus can be expressed as  $MS(\mathbf{q}^d, \mathbf{q}^s) \equiv \sum_{n=1}^N MS^n(q^{n,d}, q^{n,s})$ , which Wu et al. (1996) called the merchandizing surplus.

We can now express the social welfare as:

$$W(\mathbf{q}^d, \mathbf{q}^s) \equiv CS(\mathbf{q}^d) + \Pi(\mathbf{q}^s) + MS(\mathbf{q}^d, \mathbf{q}^s). \quad (1)$$

## 2.2 Power Flow Equations

We consider the DC load flow approximation (see, for example, Schweppe et al. 1988, and Leautier 2000 for details).  $F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$  is the power flow on transmission line  $l$  for  $l=1, \dots, L$ . We can express  $F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$  as a linear function of the net injection  $Q^n(q^{n,d}, q^{n,s}) \equiv q^{n,s} - q^{n,d}$ :

$$F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k}) \equiv \sum_{n=1}^{N-1} h^{l,n}(\mathbf{k}) Q^n(q^{n,d}, q^{n,s}), \quad l=1, \dots, L, \quad (2)$$

where  $h^{l,n}(\mathbf{k})$  is the power transfer distribution factor, or PTDF.<sup>4</sup> We ignore the transmission losses for simplicity.

$h^{l,n}(\mathbf{k})$  represents the increase in the power flow on line  $l$  resulting from a unit increase in power transferred from node  $n$  to the swing bus (without loss of generality, we can choose the swing bus to be node  $N$ ). The PTDF is determined by the physical characteristics of the network—especially the transmission capacity  $\mathbf{k}$ . Hence,  $h^{l,n}(\mathbf{k})$  can be written as a function of the capacity  $\mathbf{k}$ . Moreover, the power flow  $F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$  can also be expressed as a function of the capacity  $\mathbf{k}$ .

It should be noted that the PTDF  $h^{l,n}(\mathbf{k})$  depends on the capacity not only of line  $l$ , but also of all other lines. Similarly, note also that the power flow  $F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$  depends on the capacity of all lines. If we increase the capacity of line  $l$ , the power flow on another line  $m$ , i.e.,  $F^m(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$  may change through changes in the PTDFs.<sup>5</sup> Therefore, expansion of the

<sup>4</sup> As is customary in the electric power engineering literature, the power flow can be expressed as a function of only  $N-1$  independent net injections. The node which is not represented is often called the swing bus.

<sup>5</sup> If we change the capacity of a certain line, the admittance of this line is also modified. Then, this may cause changes in all the PTDFs, and hence cause changes in the power flows on all lines.

transmission network may have externalities that are governed by physical laws. We will discuss this issue further in the next section.

### 2.3 Nodal Pricing

Since the total power generated during any given hour has to be equal to the total amount demanded, the energy balance constraint can be written as  $Q(\mathbf{q}^d, \mathbf{q}^s) \equiv \sum_{n=1}^N Q^n(q^{n,d}, q^{n,s}) = 0$ . Moreover, the transmission capacity constraint can be expressed as  $F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k}) \leq k^l$  because the power flow on each line cannot exceed the line's capacity. We mainly consider the thermal limits of each line for the transmission capacity.

An efficient power market is characterized by the maximization of the social welfare, subject to the energy balance and transmission capacity constraints:

$$\begin{aligned} v(\mathbf{k}) &\equiv \max_{\mathbf{q}^d, \mathbf{q}^s} : W(\mathbf{q}^d, \mathbf{q}^s) & (3) \\ \text{s.t. } Q(\mathbf{q}^d, \mathbf{q}^s) &= 0 \\ F^l(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k}) &\leq k^l, \quad l = 1, \dots, L. \end{aligned}$$

Let  $\lambda$  and  $\eta^l \geq 0$  be the shadow prices associated with the energy balance and transmission capacity constraints, respectively. Then, the first-order conditions yield standard nodal pricing formulas:  $p^n = \lambda(\mathbf{k}) - \sum_{l=1}^L \eta^l(\mathbf{k}) h^{l,n}(\mathbf{k})$  for  $n = 1, \dots, N-1$ ;  $p^n = \lambda(\mathbf{k})$  for the swing bus  $n = N$ . Let  $\mathbf{q}^*(\mathbf{k}) \equiv (\mathbf{q}^{d*}(\mathbf{k}), \mathbf{q}^{s*}(\mathbf{k}))'$  denote the optimal power demand and supply at nodes under nodal pricing.  $v(\mathbf{k}) \equiv W(\mathbf{q}^{d*}(\mathbf{k}), \mathbf{q}^{s*}(\mathbf{k}))$  represents the maximum social welfare under nodal pricing, or, simply, the optimal value function. Note that  $v(\mathbf{k})$  is the social welfare in the short run in which the transmission capacity  $\mathbf{k}$  is given and fixed. See Bohn et al. (1984) and Schweppe et al. (1988) for the details of nodal pricing.

Note that  $\eta^l(\mathbf{k})$  represents the shadow congestion price of line  $l$ . Hence, the Transco obtains the congestion rent  $\rho(\mathbf{k}) \equiv \sum_{l=1}^L \eta^l(\mathbf{k}) k^l$  under nodal pricing. Wu et al. (1996) show that the congestion rent is equal to the merchandizing surplus at the optimal dispatch:

$$MS(\mathbf{q}^{d*}(\mathbf{k}), \mathbf{q}^{s*}(\mathbf{k})) = \rho(\mathbf{k}). \quad (4)$$

### 3. Optimal Transmission Capacity under Nodal Pricing

In the long run, optimal expansion of the transmission network is critical for an efficient power market to develop. It should also be noted that the total cost of transmission facilities must be recovered in an appropriate way. The Transco must earn enough revenue in a power market to cover its capital and other costs when government subsidies are not available. In this section, we first derive the optimal transmission capacity, considering the Transco's budget constraint. We then discuss the optimal transmission capacity without the budget constraint, assuming that the regulator can subsidize the Transco for the deficit. Let us start by defining some key terms associated with costs, externalities, and congestion rent.

Let  $c(\mathbf{k})$  denote the capacity cost of transmission lines, mainly the fixed capital cost. We assume that  $c(\mathbf{k})$  is twice continuously differentiable and a non-decreasing concave function,

considering economies of scale in network expansion. Note that  $c_l(\mathbf{k}) \equiv \partial c(\mathbf{k})/\partial k^l$  is the marginal capacity cost of line  $l$ .<sup>6</sup> Subtracting the capacity cost from the congestion rent yields the Transco's long-run profit  $\sigma(\mathbf{k}) \equiv \rho(\mathbf{k}) - c(\mathbf{k})$  under nodal pricing.

As we have seen in the previous section, an increase in the capacity of line  $l$  may modify the PTDfS, and hence cause changes in the power flow on another line  $m$ , i.e.,  $F^m(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$ , even if the injections and withdrawals of all nodes are kept unchanged. This phenomenon is caused by the technological effects of network expansion, which is governed by physical laws.

Let us evaluate the technological effects of capacity expansion on the power flows.  $F_l^m(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k}) \equiv \partial F^m(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})/\partial k^l$  denotes a marginal change in the power flow on line  $m$  when we increase the capacity of line  $l$  by one unit, keeping all the injections and withdrawals unchanged. Thus,  $\eta^m(\mathbf{k})F_l^m(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})$  can be interpreted as the value of a marginal change in the power flow on line  $m$ , as measured by the shadow congestion price of this line. We can now define  $\phi^l(\mathbf{k})$  as (marginal) *externalities associated with power flow changes* by considering the effects on all lines:

$$\phi^l(\mathbf{k}) \equiv \sum_{m=1}^L \eta^m(\mathbf{k})F_l^m(\mathbf{q}^d, \mathbf{q}^s, \mathbf{k})|_{\mathbf{q}=\mathbf{q}^*(\mathbf{k})}, \quad l = 1, \dots, L. \quad (5)$$

That is,  $\phi^l(\mathbf{k})$  is the value of marginal changes in the power flows as measured by the shadow congestion prices, which is associated with a unit increase in the line  $l$ 's capacity. Note that  $\phi^l(\mathbf{k})$  is evaluated at the optimal dispatch, i.e., at  $\mathbf{q}^*(\mathbf{k})$ .

$\phi^l(\mathbf{k})$  has essentially the same structure as what Leautier (2000) simply calls an indirect effect (externalities among transmission lines). However, the difference is that we explicitly define  $\phi^l(\mathbf{k}) > 0$  as *negative* externalities. Since  $v_l(\mathbf{k}) = \eta^l(\mathbf{k}) - \phi^l(\mathbf{k})$  holds from the envelope theorem, the additional capacity of line  $l$  increases the social welfare by  $\eta^l(\mathbf{k})$ , whereas it has the effect of decreasing the social welfare by  $\phi^l(\mathbf{k})$  at the same time. Thus,  $\phi^l(\mathbf{k}) > 0$  can be interpreted as the marginal externality cost, and hence considered to be negative externalities.<sup>7</sup>

We can then define *the social marginal capacity cost*  $\tau^l(\mathbf{k})$  by adding the marginal externality cost  $\phi^l(\mathbf{k})$  to the (private) marginal capacity cost  $c_l(\mathbf{k})$ :

$$\tau^l(\mathbf{k}) \equiv c_l(\mathbf{k}) + \phi^l(\mathbf{k}), \quad l = 1, \dots, L. \quad (6)$$

Next, we consider the effects of capacity expansion on the congestion rent. A unit increase in the capacity of line  $l$  gives the Transco an extra congestion rent  $\eta^l(\mathbf{k})$  on this additional unit. Furthermore, it should be noted that an additional unit will cause changes in the congestion rent on the *inframarginal* transmission units, that is,  $\eta_l^m(\mathbf{k})k^m$  for  $m = 1, \dots, L$ . We then

<sup>6</sup> A functional symbol with a subscript will indicate the derivative of the original function with respect to the variable denoted by the superscript.

<sup>7</sup>  $\phi^l(\mathbf{k}) < 0$  yields the marginal externality benefit, and hence is considered to be positive externalities.



define  $\psi^l(\mathbf{k})$  as (marginal) *changes in the congestion rent on the inframarginal capacity*:<sup>8</sup>

$$\psi^l(\mathbf{k}) \equiv -\sum_{m=1}^L \eta_l^m(\mathbf{k}) k^m, \quad l=1, \dots, L. \quad (7)$$

It is straightforward that  $\rho_l(\mathbf{k}) = \eta^l(\mathbf{k}) - \psi^l(\mathbf{k})$  holds.

Let  $u(\mathbf{k})$  denotes the social welfare in the long run. Note that  $u(\mathbf{k}) \equiv v(\mathbf{k}) - c(\mathbf{k}) \equiv CS(\mathbf{q}^{d*}(\mathbf{k})) + \Pi(\mathbf{q}^{s*}(\mathbf{k})) + \sigma(\mathbf{k})$ . We now represent the welfare maximization problem, considering the Transco's budget constraint:

$$\begin{aligned} \max_{\mathbf{k}} : u(\mathbf{k}) \\ \text{s.t. } \sigma(\mathbf{k}) \geq 0. \end{aligned} \quad (8)$$

Let  $\xi \geq 0$  be the shadow price associated with the budget constraint, and let  $R \equiv \xi/(1+\xi)$ . We then obtain the optimal transmission capacity  $\mathbf{k}^*$ , taking into account the Transco's budget constraint:

**Proposition 1:** *The optimal transmission capacity under a budget constraint is such that the deviation of the congestion price from the social marginal capacity cost is proportional to the difference between changes in the congestion rent on the inframarginal capacity and externalities associated with power flow changes. That is,  $\mathbf{k}^*$  satisfies*

$$\eta^l(\mathbf{k}) - \tau^l(\mathbf{k}) = R \{ \psi^l(\mathbf{k}) - \phi^l(\mathbf{k}) \}, \quad l=1, \dots, L. \quad (9)$$

The proof is given in the appendix. In the long run, the congestion rent under nodal pricing will change in accordance with the increase or decrease in the transmission capacity. Therefore, by adjusting the transmission capacity appropriately, a linear congestion charge can be set in such a way that the congestion rent would recover the capacity cost of transmission lines. Proposition 1 shows that, under nodal pricing, the transmission capacity should be adjusted on the basis of both *changes in the congestion rent on the inframarginal capacity* and *externalities associated with power flow changes* in order to increase the long-run social welfare, while satisfying the budget constraint. This rule can be regarded as a kind of the Ramsey rule, which is associated with not the power  $\mathbf{q}$ , but the transmission capacity  $\mathbf{k}$ . In other words, it determines the second-best investment level  $\mathbf{k}^*$ , while nodal prices are charged for the power  $\mathbf{q}^*$ .

If the regulator can subsidize the Transco for the deficit, we will attain the optimal transmission capacity without the Transco's budget constraint. The first-best transmission capacity, denoted by  $\mathbf{k}^f$ , is straightforward from Proposition 1. Substituting 0 for  $R$  yields

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<sup>8</sup> Suppose a simple two-node case in which there is only one transmission line. If the line's capacity is increased, the congestion price is decreased. Hence, the congestion rent on the inframarginal transmission units is reduced, that is,  $\eta'(k)k$  is negative. We can evaluate  $\eta'(k)k$  in absolute value by defining  $\psi(k)$  with a negative sign. Similarly, we define  $\psi^l(\mathbf{k})$  with a negative sign.

the following corollary:

**Corollary 1:** (Leautier 2000) *The optimal transmission capacity without a budget constraint is such that the congestion price is equal to the social marginal capacity cost for each line. That is,  $\mathbf{k}^f$  satisfies*

$$\eta^l(\mathbf{k}) = \tau^l(\mathbf{k}), \quad l = 1, \dots, L. \quad (10)$$

This corollary restates Corollary 1 of Leautier (2000) in view of *the social marginal capacity cost* which Leautier did not explicitly define. To put it another way, the first-best capacity expansion should be determined by considering the social marginal capacity cost that incorporates externalities of capacity expansion.

#### 4. Extended Price Cap regulation for Transco

Without appropriate regulation, the monopoly Transco will attempt to maximize its long-run profit  $\sigma(\mathbf{k})$  instead of the long-run social welfare  $u(\mathbf{k})$ . Thus, the Transco will determine the transmission capacity in such a way that  $\eta^l(\mathbf{k}) = c_l(\mathbf{k}) + \psi^l(\mathbf{k})$  holds for each line. As a result, the attained capacity will deviate from the optimal capacity with and without the budget constraint, derived in section 3.

In section 4 and 5, we examine regulatory incentive mechanisms for efficient investment in the transmission network, taking into account both technological externalities among transmission lines and information asymmetry between the regulator and the Transco. Special attention is focused on developing incentive mechanisms that attempt to internalize technological externalities governed by physical laws. Moreover, we focus on asymmetric information about the Transco's cost structure, supposing that the regulator does not know the capacity cost function of transmission lines, while the regulator can observe the actual cost (not function) in each period.

In this section, two forms of price cap regulation that can avoid government subsidy are developed to attain the optimal transmission capacity under the budget constraint. In section 5, a surplus-based scheme with government transfers will be examined to achieve the optimal transmission capacity without the budget constraint.

##### 4.1 Extended Price Cap Mechanism for Transco

Price caps are well-known as one of powerful incentive mechanisms both in theory and in practice. In general, a price cap mechanism sets some ceiling for prices to be charged by the regulated firm. The firm is allowed to choose any prices as long as some average price index is below the ceiling, i.e., price cap.

A typical price cap mechanism takes a simple form of a constraint on prices, expressed generally as  $\mathbf{P}^{t'} \cdot \mathbf{Q}^{t-1} \leq \mathbf{P}^{t-1} \cdot \mathbf{Q}^{t-1}$ , where  $\mathbf{P}$  and  $\mathbf{Q}$  are price and quantity vectors of some goods, respectively. In other words, the firm is allowed to choose any prices as long as a Laspeyres price index,  $\mathbf{P}^{t'} \cdot \mathbf{Q}^{t-1} / \mathbf{P}^{t-1} \cdot \mathbf{Q}^{t-1}$ , is not greater than a price cap, 1. It is shown that prices will converge to Ramsey prices in any long-run stationary equilibrium. Note that the regulator need not have any knowledge of the firm's cost function. See, for example, Vogelsang

(1989) and Armstrong et al. (1994), for further discussion.

It may be natural to consider some form of cap on the congestion prices in order to induce efficient capacity expansion of transmission lines. Such a mechanism might take a simple form of a constraint on the congestion prices,  $\boldsymbol{\eta}(\mathbf{k}^t)' \cdot \mathbf{k}^{t-1} \leq \boldsymbol{\eta}(\mathbf{k}^{t-1})' \cdot \mathbf{k}^{t-1} \equiv \rho(\mathbf{k}^{t-1})$ , which would be a straightforward application of a standard price cap mechanism. That is, the Transco might be allowed to choose the current period's capacity  $\mathbf{k}^t$ , and hence the congestion prices  $\boldsymbol{\eta}(\mathbf{k}^t)$  as long as a Laspeyres congestion price index,  $\boldsymbol{\eta}(\mathbf{k}^t)' \cdot \mathbf{k}^{t-1} / \boldsymbol{\eta}(\mathbf{k}^{t-1})' \cdot \mathbf{k}^{t-1}$ , would not be greater than a price cap, 1.

However, unfortunately, a typical price cap mechanism cannot be directly applied to induce optimal expansion of the transmission network. First, a typical price cap will not achieve the optimal transmission capacity under the Transco's budget constraint in the presence of technological externalities among transmission lines. Since technological externalities are not taken into consideration under a standard mechanism, the long-run stationary capacity, if any, will deviate from the optimal capacity derived in Proposition 1. Second, and related to the first point, the convergence of the process may not be assured. The convexity of the consumers' surplus with respect to prices assures the convergence of the process under a typical price cap. However, the convexity of the consumers' surplus with respect to the congestion prices would be ambiguous.

Therefore, we have to modify and extend the original price cap mechanism in order to internalize technological externalities and assure the convergence of the process. By introducing an additional constraint on the capacity, we now define *the extended price cap mechanism for Transco* as follows:

**Definition 1:** (*The extended price cap mechanism for Transco*) In each period  $t$ , the regulator allows the Transco to choose the transmission capacity  $\mathbf{k}^t$  that satisfies the following constraint:

$$\boldsymbol{\eta}(\mathbf{k}^t)' \cdot \mathbf{k}^{t-1} + \boldsymbol{\eta}(\mathbf{k}^{t-1})' \cdot \mathbf{F}(\mathbf{q}^{t-1}, \mathbf{k}^t) \leq 2\rho(\mathbf{k}^{t-1}), \quad (11)$$

$$cs(\mathbf{k}^t) \geq cs(\mathbf{k}^{t-1}). \quad (12)$$

The second term of the LHS of the first inequality,  $\boldsymbol{\eta}(\mathbf{k}^{t-1})' \cdot \mathbf{F}(\mathbf{q}^{t-1}, \mathbf{k}^t)$ , is the extra element added to the original price cap mechanism. This new element has a crucial effect in internalizing technological externalities. It should be noted that  $\mathbf{F}(\mathbf{q}^{t-1}, \mathbf{k}^t)$  represents the power flow under the current period's capacity  $\mathbf{k}^t$  if it *were* applied to the previous period's power transaction at nodes under nodal pricing, namely  $\mathbf{q}^{t-1} \equiv \mathbf{q}^*(\mathbf{k}^{t-1})$ . Note also that the RHS of the first inequality is doubled. The second inequality assures the convergence of the process by preventing the Transco from reducing the consumers' surplus. Note that  $cs(\mathbf{k}) \equiv CS(\mathbf{q}^*(\mathbf{k}))$ .

We suppose that both the regulator and the Transco can observe the information needed to implement our new mechanism. The congestion prices  $\boldsymbol{\eta}$  and the congestion rent  $\rho$  are derived in the process of calculating nodal prices. The power flow  $\mathbf{F}$  can also be calculated by the software that uses a standard technique in power system engineering. Fortunately, the consumers' surplus  $cs$  can also be derived in the process of calculating nodal prices. It should be noted that both the regulator and the Transco can use the information obtained in the spot market; that is, the demand bid curve (function) and the supply offer curve (function). Note also that the regulator needs to know neither the Transco's capacity cost function nor the actual cost.

We now establish the following proposition.

**Proposition 2:** *For any given  $\sigma(\mathbf{k}^0) \geq 0$ , the extended price cap mechanism induces the Transco to choose the optimal transmission capacity under a budget constraint, i.e.,  $\mathbf{k}^*$ , in a dynamic process. That is, there exists  $\mathbf{k}^*$  such that:*

$$(i) \quad \lim_{t \rightarrow \infty} u(\mathbf{k}^t) = u(\mathbf{k}^*). \quad (13)$$

$$(ii) \quad \boldsymbol{\eta}(\mathbf{k}^*) - \boldsymbol{\tau}(\mathbf{k}^*) = R \left\{ \boldsymbol{\psi}(\mathbf{k}^*) - \boldsymbol{\phi}(\mathbf{k}^*) \right\}. \quad (14)$$

The proof is given in the appendix. Here, we attempt to provide an intuitive explanation of this mechanism. If the Transco earned a positive profit in period  $t-1$ , it can, if it wants, earn the same positive profit by choosing  $\mathbf{k}^{t-1}$  in period  $t$ , without reducing the consumers' surplus. In general, the Transco will choose a different capacity vector in period  $t$  and be strictly better off. Generally, the consumers' surplus can also rise over time. Since we consider a fully competitive generation market, the long-run economic profit of generators is supposed to be zero. Overall, we get a monotonically increasing sequence in the social welfare, and the sequence is shown to converge to the optimal transmission capacity under the Transco's budget constraint.

It should be emphasized that our new mechanism can internalize technological externalities among transmission lines, whereas a standard mechanism cannot. The basic idea underlying this mechanism is that the gradient vector of  $\boldsymbol{\eta}(\mathbf{k}^{t-1}) \cdot \mathbf{F}(\mathbf{q}^{t-1}, \mathbf{k}^t)$  with respect to  $\mathbf{k}^t$ , which is a key component, becomes the vector  $\boldsymbol{\phi}$  in the limit; that is, externalities associated with power flow changes. As a result, the three surfaces, namely, the iso-welfare surface, the iso-profit surface, and the regulatory constraint surface are tangent to each other at the limit point, which coincides with  $\mathbf{k}^*$ . Hence, we obtain the optimal transmission capacity under the budget constraint. The extended price cap mechanism would provide new insights into incentive regulation for the Transco in a competitive power market.

## 4.2 Extended Vogelsang-Finsinger Mechanism for Transco

The Vogelsang and Finsinger mechanism, called the V-F mechanism, is a dynamic scheme that induces the monopoly firm to move, over time, to Ramsey prices, where the firm's profit is zero. The original V-F mechanism takes a simple form of a constraint on prices, expressed as  $\mathbf{P}^t \cdot \mathbf{Q}^{t-1} \leq C(\mathbf{Q}^{t-1})$ . In other words, the firm is allowed to charge prices in the current period that would not result in any positive profit if they *were* applied to the previous period's quantities and cost. Under some assumptions, Vogelsang and Finsinger (1979) show that prices will converge to Ramsey prices and the firm's profit will vanish in the limit. The regulator using the V-F mechanism need not know the firm's cost function  $C(\cdot)$ , but only the reported cost in the previous period,  $C(\mathbf{Q}^{t-1})$ . The V-F mechanism is different from a typical price cap mechanism in that the RHS of the inequality is the reported cost  $C(\mathbf{Q}^{t-1})$ , instead of the revenue  $\mathbf{P}^{t-1} \cdot \mathbf{Q}^{t-1}$ . If the previous period's profit is positive, the constraint can be expressed as  $\mathbf{P}^t \cdot \mathbf{Q}^{t-1} \leq C(\mathbf{Q}^{t-1}) \leq \mathbf{P}^{t-1} \cdot \mathbf{Q}^{t-1}$ . Thus, the constraint under the V-F mechanism is tighter than that under a typical price cap mechanism.

Similar to the argument in the previous subsection, we have to modify and extend the original V-F mechanism in the presence of technological externalities. We define *the extended Vogelsang-Finsinger (V-F) mechanism for Transco* as follows:

**Definition 2:** (*The extended V-F mechanism for Transco*) In each period  $t$ , the regulator allows the Transco to choose the transmission capacity  $\mathbf{k}^t$  that satisfies the following constraint:

$$\boldsymbol{\eta}(\mathbf{k}^t)' \cdot \mathbf{k}^{t-1} + \boldsymbol{\eta}(\mathbf{k}^{t-1})' \cdot \mathbf{F}(\mathbf{q}^{t-1}, \mathbf{k}^t) \leq c(\mathbf{k}^{t-1}) + \rho(\mathbf{k}^{t-1}). \quad (15)$$

$$cs(\mathbf{k}^t) \geq cs(\mathbf{k}^{t-1}). \quad (16)$$

The second term of the LHS of the first inequality is  $c(\mathbf{k}^{t-1}) + \rho(\mathbf{k}^{t-1})$  instead of  $2\rho(\mathbf{k}^{t-1})$ , which is the only difference between this mechanism and the extended price cap mechanism. Note that, in general, the constraint under the extended V-F mechanism is tighter than that under the extended price cap mechanism.

We focus on asymmetric information about the Transco's cost structure in a similar way as the original V-F mechanism. That is, we suppose that the regulator does not know the Transco's capacity cost *function*  $c(\cdot)$ , but can observe the actual cost (not function) in the previous period,  $c(\mathbf{k}^{t-1})$ . Similar to the previous subsection, the congestion prices  $\boldsymbol{\eta}$ , the congestion rent  $\rho$ , and the consumers' surplus  $cs$  can be derived in the process of calculating nodal prices. The power flow  $\mathbf{F}$  can also be calculated by the software that uses a standard technique in power system engineering.

The assumption imposed on the cost function under the original V-F mechanism is also applied; that is, the cost function  $c(\mathbf{k})$  exhibits no decreasing return to scale, which comes from economies of scale in network expansion. Moreover, we impose some natural assumptions associated with the nature of the power transaction. First, from the standpoint of physics, the power transfer distribution factor  $h^{l,n}(\mathbf{k})$  is assumed to be homogenous of degree 0. To put it another way,  $\mathbf{h}(\mathbf{k})$  will remain the same if  $\mathbf{k}$  is increased by a uniform percentage. Hence, the power flow function  $\mathbf{F}(\mathbf{q}, \mathbf{k})$  is assumed to be homogenous of degree 0 with respect to  $\mathbf{k}$ ; that is, the way how electric power is distributed physically on each line will remain the same if the capacity is scaled up proportionally, keeping all the injections and withdrawals unchanged. Second, from an economic viewpoint, the congestion of each line will, in general, tend to be relieved as the entire transmission capacity constraint becomes less severe. In other words, the congestion prices  $\boldsymbol{\eta}(\mathbf{k})$  is supposed to fall as the entire transmission capacity  $\mathbf{k}$  becomes larger and larger. Thus, we assume that  $\lim_{\alpha \rightarrow \infty} \boldsymbol{\eta}(\alpha \mathbf{k}) = \mathbf{0}$  for  $\alpha > 1$ . Third, and related to the second point, we assume that  $cs(\alpha \mathbf{k}) \geq cs(\mathbf{k})$  for  $\alpha > 1$ . As the entire transmission capacity constraint becomes less severe, consumers can, in general, buy more electric power produced by cheaper generators.

We now represent the following proposition.

**Proposition 3:** For any given  $\sigma(\mathbf{k}^0) \geq 0$ , the E-V-F mechanism induces the Transco to choose the optimal transmission capacity under a budget constraint, i.e.,  $\mathbf{k}^*$ , in a dynamic process. The Transco's profit is then zero. That is, there exists  $\mathbf{k}^*$  such that:

$$(i) \quad \lim_{t \rightarrow \infty} cs(\mathbf{k}^t) = cs(\mathbf{k}^*). \quad (17)$$

$$(ii) \quad \sigma(\mathbf{k}^*) = 0. \quad (18)$$

$$(iii) \quad \eta(\mathbf{k}^*) - \tau(\mathbf{k}^*) = R \left\{ \psi(\mathbf{k}^*) - \phi(\mathbf{k}^*) \right\}. \quad (19)$$

The proof is given in the appendix. The extended V-F mechanism is similar to the original V-F mechanism in that the regulatory constraint is tightened every time the Transco makes a profit, and in turn, every time the regulatory constraint is tightened, the welfare increases. Thus, we can show that the monotonically increasing sequence in the welfare will converge and the Transco's profit will vanish in the limit.

The extended V-F mechanism has a desirable property that the Transco will earn no profit in the limit. By contrast, the extended price cap mechanism may not fully eliminate the profit over time since it does not require the Transco to return the previous profit to consumers.

However, the extended V-F mechanism has a severe problem of strategic waste, similar to the original V-F mechanism. As Sappington (1980) demonstrates, the firm can have an incentive to indulge in wasteful expenditures in order to relax future constraints. In contrast, the extended price cap mechanism can encourage productive efficiency since the cost is not mentioned in the constraint. Therefore, the extended price cap mechanism has a desirable property that the Transco will never engage in strategic waste.<sup>9</sup>

## 5. Incremental Surplus Subsidy Scheme for Transco

In most cases, the regulator may not directly compensate the regulated monopoly Transco for the deficit. However, if the regulator can subsidize the Transco, the surplus-based schemes can be used in order to induce the Transco to choose the optimal transmission capacity without its budget constraint. One such scheme is a dynamic regulatory mechanism proposed by Sappington and Sibley (1988). Their mechanism, called the incremental surplus subsidy (ISS) scheme, provides a monopoly firm with a subsidy based on a period-to-period change in the consumer surplus (subtracted by the firm's operating profit). They demonstrate that the ISS scheme can eventually induce the monopoly firm to maximize the total social surplus, and hence achieve the first-best outcome; namely, marginal cost pricing with no waste and zero profit. It should be noted that the result can be attained without the regulator knowing the cost function of the monopoly firm.

The market they have investigated is a standard monopoly market, where a monopoly firm simply produces and sells products to consumers. By contrast, a power market has a more complicated vertical structure; that is, the regulated Transco, which is an upstream monopolist,

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<sup>9</sup> Formally, the extended V-F mechanism assumes that the Transco is myopic, i.e., its discount factor is zero. By contrast, the Transco can be a non-myopic firm that maximizes the discounted value of its profits under the extended price cap mechanism.

produces the transmission capacity  $\mathbf{k}$  and earns the congestion rent  $\rho(\mathbf{k})$ ; generators, which are downstream competitive firms, generate electric power  $\mathbf{q}^*(\mathbf{k})$  and sell it to consumers through the transmission lines. In spite of this market structure, we can directly apply the ISS scheme to the regulation of the Transco in a competitive power market.

Although Gans and King (2000), and Joskow and Tirole (2002) suggest implementing the surplus-based schemes to induce efficient transmission investment, they do not explicitly formulate the mechanism on the basis of the original ISS scheme formula which Sappington and Sibley have derived. In contrast, our model framework enables explicit and straightforward application of the original ISS scheme formula to the regulation of the Transco.

Based on the notation of the original ISS scheme, let  $e(\mathbf{k}, w) \equiv c(\mathbf{k}) + w$  denote the Transco's expenditure for the capacity, where  $w \geq 0$  is a possible waste. Then, define the Transco's profit by subtracting the total expenditure  $e(\mathbf{k}, w)$  from the congestion rent  $\rho(\mathbf{k})$ .  $\rho(\mathbf{k}) - e(\mathbf{k}, w)$  may differ from  $\sigma(\mathbf{k}) \equiv \rho(\mathbf{k}) - c(\mathbf{k})$  by the possible waste. We assume that the regulator does not know the capacity cost function  $c(\cdot)$  and the possible waste  $w$ ; and hence the expenditure function  $e(\cdot)$ , whereas the Transco has these information. On the other hand, The regulator can observe the reported (audited) expenditure in the previous period,  $e(\mathbf{k}^{t-1}, w^{t-1})$ . We assume that both the regulator and the Transco have the same information about the consumers' surplus  $cs$  and the congestion rent  $\rho$ , which are derived in the process of calculating nodal prices.<sup>10</sup> Note that the long-run economic profit of generators will be zero as in the previous section.

We can then define the incremental surplus subsidy (ISS) scheme for Transco, which is the direct application of the original ISS scheme, as follows:

**Definition 3:** (*The ISS scheme for Transco*) In each period  $t$ , the regulator allows the Transco to choose the transmission capacity  $\mathbf{k}^t$ , and provides the following subsidy  $s^t$  to the Transco for  $t = 0, \dots, \infty$ :

$$s^t \equiv \{cs(\mathbf{k}^t) - cs(\mathbf{k}^{t-1})\} - \{\rho(\mathbf{k}^{t-1}) - e(\mathbf{k}^{t-1}, w^{t-1})\}. \quad (20)$$

The first term of the RHS represents the increment in the consumers' surplus, generated by the capacity change between the current period  $t$  and the previous period  $t-1$ . The second term is the Transco's profit in the previous period  $t-1$ , calculated on the basis of the reported expenditure  $e(\mathbf{k}^{t-1}, w^{t-1})$ . Note again that the expenditure function itself is known, not to the regulator, but to the Transco. Taken together, the subsidy under the ISS scheme for Transco can be expressed as the improvement in the consumers' surplus subtracted by the Transco's lagged profit.

Clearly, the basic idea of Definition 3 is the same as that of the original ISS scheme. Thus, it is obvious that we can obtain the same result as under the ISS scheme.

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<sup>10</sup> We suppose that both the regulator and the Transco can use the information obtained in the spot market: that is, the demand bid curve (function) and the supply offer curve (function). When we assume that these information is not fully available, the scheme proposed by Finsinger and Vogelsang (1985), which does not utilize the demand and supply curves (functions), may be a substitute for the ISS scheme. However, it may take more periods before the first-best outcome is attained.

**Proposition 4:** *Under the ISS scheme for Transco, the followings hold for any given  $s^0$ ,  $\mathbf{k}^0$ :*

- (i) *The Transco chooses the optimal transmission capacity without a budget constraint, i.e.,  $\mathbf{k}^f$  in every period:*

$$\eta^l(\mathbf{k}^f) = \tau^l(\mathbf{k}^f), \quad l = 1, \dots, L, \quad t = 1, \dots, \infty. \quad (21)$$

- (ii) *The Transco operates at minimum cost in every period:*

$$w^t = 0, \quad t = 0, \dots, \infty. \quad (22)$$

- (iii) *The Transco gains no profit from the second period on (i.e., the Transco can gain strictly positive profit only in the first period):*

$$\rho(\mathbf{k}^t) - e(\mathbf{k}^t, w^t) + s^t = 0, \quad t = 2, \dots, \infty. \quad (23)$$

The proof is omitted because it is analogous to that of Proposition 1 in Sappington and Sibley (1988).<sup>11</sup> It should be noted that the ISS scheme for Transco assures the optimal transmission capacity without a budget constraint, internalizing technological externalities among transmission lines. Furthermore, the desirable features of the original ISS scheme such as inducing no waste hold true.

## 6. Concluding Remarks

This paper has examined regulatory incentive mechanisms for efficient investment in the transmission network, taking into account both technological externalities among transmission lines and information asymmetry between the regulator and the transmission company (Transco). First, by adding extra constraints associated with the power flow, we have developed an extended price cap mechanism that can internalize technological externalities among transmission lines. We have shown that this new mechanism induces the Transco to choose the optimal transmission capacity under its budget constraint. An extended form of the Vogelsang and Finsinger (V-F) mechanism has also been introduced. Next, we have examined the surplus-based scheme with government transfers. We have provided a formal analysis of the incremental surplus subsidy (ISS) scheme specifically for the Transco, demonstrating that it induces the Transco to choose the optimal transmission capacity without the budget constraint.

Linear tariffs for transmission services have been examined on the basis of the extended forms of the price cap and V-F mechanism. In practice, we may, in some way, combine the linear tariffs of these types and the two-part tariff that is based on Vogelsang's (2001) scheme. With regard to the transmission constraints, we have focused on thermal limit of each line. Further work is needed to incorporate other realistic limits such as voltage limit and stability limit. Although we have considered a perfectly competitive wholesale market, the market power of generators has become a relevant issue. Another track for future research is to extend the framework of the incentive regulation approach to incorporate an imperfectly competitive market.

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<sup>11</sup> A complete proof is available upon request.



## Appendix

### Proof of Proposition 1

Define the Lagrangian as

$$L \equiv v(\mathbf{k}) - c(\mathbf{k}) + \xi \{ \rho(\mathbf{k}) - c(\mathbf{k}) \}. \quad (24)$$

Then, the first-order condition with respect to  $k^l$  yields

$$\begin{aligned} v_l(\mathbf{k}) - c_l(\mathbf{k}) + \xi \{ \rho_l(\mathbf{k}) - c_l(\mathbf{k}) \} &= 0 \\ \eta^l(\mathbf{k}) - \phi^l(\mathbf{k}) - c_l(\mathbf{k}) + \xi \{ \eta^l(\mathbf{k}) - \psi^l(\mathbf{k}) - c_l(\mathbf{k}) \} &= 0 \\ (1 + \xi) [ \eta^l(\mathbf{k}) - \{ \phi^l(\mathbf{k}) + c_l(\mathbf{k}) \} ] &= \xi \{ \psi^l(\mathbf{k}) - \phi^l(\mathbf{k}) \}. \end{aligned} \quad (25)$$

Hence, we have

$$\eta^l(\mathbf{k}) - \tau^l(\mathbf{k}) = R \{ \psi^l(\mathbf{k}) - \phi^l(\mathbf{k}) \} \quad (26)$$

for  $l = 1, \dots, L$ . ■

### Proof of Proposition 2

We simply write  $\mathbf{k}$  for  $\mathbf{k}^t$ , which is the decision variable in period  $t$ . We also write  $\bar{\mathbf{k}}$  for  $\mathbf{k}^{t-1}$ .  $\bar{\mathbf{k}}$  is a constant in period  $t$  since it is the decision variable in the previous period  $t-1$ . In addition, we write  $\bar{\mathbf{q}}$  for  $\mathbf{q}^{t-1}$ . Furthermore, let  $\delta(\mathbf{k}) \equiv \boldsymbol{\eta}(\mathbf{k}) \cdot \bar{\mathbf{k}} + \boldsymbol{\eta}(\bar{\mathbf{k}}) \cdot \mathbf{F}(\bar{\mathbf{q}}, \mathbf{k})$ . Then, the first inequality of the extended price cap mechanism can be expressed as  $\delta(\mathbf{k}) \leq 2\rho(\bar{\mathbf{k}})$ . Basically, we give a proof for a myopic case for a comparison with the case of the extended V-F mechanism in Proposition 3.

We first show that the extended price cap mechanism is feasible for the Transco. Suppose that the Transco makes a profit in the previous period  $t-1$ , that is,  $\sigma(\bar{\mathbf{k}}) = \rho(\bar{\mathbf{k}}) - c(\bar{\mathbf{k}}) \geq 0$ . Since  $\delta(\bar{\mathbf{k}}) = \boldsymbol{\eta}(\bar{\mathbf{k}}) \cdot \bar{\mathbf{k}} + \boldsymbol{\eta}(\bar{\mathbf{k}}) \cdot \mathbf{F}(\bar{\mathbf{q}}, \bar{\mathbf{k}}) = 2\rho(\bar{\mathbf{k}})$  holds,  $\mathbf{k} = \bar{\mathbf{k}}$  satisfies the first inequality of the mechanism. Of course,  $\mathbf{k} = \bar{\mathbf{k}}$  also satisfies the second inequality. Thus, the Transco can, if it wants, choose  $\mathbf{k} = \bar{\mathbf{k}}$  in the current period  $t$  and make a non-negative profit. Therefore, it follows that  $\sigma(\mathbf{k}) \geq \sigma(\bar{\mathbf{k}}) \geq 0$ . Assuming that  $\sigma(\mathbf{k}^0) \geq 0$  in the first period, and by induction,  $\sigma(\mathbf{k}) \geq 0$  holds for all  $t$ . In other words, the extended price cap mechanism is feasible for the Transco in the sense that it can make a non-negative profit in all periods.

Since  $cs(\mathbf{k}) \geq cs(\bar{\mathbf{k}})$  and  $\sigma(\mathbf{k}) \geq \sigma(\bar{\mathbf{k}})$  hold under the extended price cap mechanism,  $\{u(\mathbf{k})\}_{t=0}^{\infty}$  is a non-decreasing sequence, which is bounded by a constrained welfare maximum. Hence, it converges, and the limit  $\lim_{t \rightarrow \infty} u(\mathbf{k}) = u(\hat{\mathbf{k}})$  exists. Note that the long-run social welfare is expressed as  $u(\mathbf{k}) \equiv cs(\mathbf{k}) + \sigma(\mathbf{k})$ . Here, the long-run economic profit of generators,  $\Pi(\mathbf{q}^{**}(\mathbf{k}))$ , is supposed to be zero since we consider a fully competitive generation

market. If the Transco is non-myopic and acts strategically, it may choose  $\mathbf{k}$  such that  $\sigma(\mathbf{k}) < \sigma(\bar{\mathbf{k}})$  for some periods in order to have more profitable periods later. However, such a finite sequence can always be ended at periods when the profit is greater than the past profit  $\sigma(\bar{\mathbf{k}})$ . Hence, we can get a non-decreasing sub-sequence of the Transco's profit even in a non-myopic case.

Considering the regulator's problem,  $\max_{\mathbf{k}} \{u(\mathbf{k}) \equiv cs(\mathbf{k}) + \sigma(\mathbf{k}) : \sigma(\mathbf{k}) \geq 0\}$ , we can rewrite the first-order condition as follows:

$$\nabla_{\mathbf{k}} cs(\mathbf{k}) = -(1 + \zeta) \nabla_{\mathbf{k}} \sigma(\mathbf{k}), \quad (27)$$

where  $\zeta \geq 0$  is a Lagrange multiplier.

On the other hand, the Transco's problem in each period can be expressed as  $\max_{\mathbf{k}} \{\sigma(\mathbf{k}) : \delta(\mathbf{k}) \leq 2\rho(\bar{\mathbf{k}}), cs(\mathbf{k}) \geq cs(\bar{\mathbf{k}})\}$ . Then, the first-order condition yields

$$\begin{aligned} \nabla_{\mathbf{k}} \sigma(\mathbf{k}) &= \tau \nabla_{\mathbf{k}} \delta(\mathbf{k}) - \mu \nabla_{\mathbf{k}} cs(\mathbf{k}) \\ &= \tau \{\bar{\mathbf{k}}' \cdot \nabla_{\mathbf{k}} \boldsymbol{\eta}(\mathbf{k}) + \boldsymbol{\eta}(\bar{\mathbf{k}})' \cdot \nabla_{\mathbf{k}} \mathbf{F}(\bar{\mathbf{q}}, \mathbf{k})\} - \mu \nabla_{\mathbf{k}} cs(\mathbf{k}), \end{aligned} \quad (28)$$

where  $\tau \equiv \tau' \geq 0$  and  $\mu \equiv \mu' \geq 0$  are Lagrange multipliers. Thus, the following condition holds in the limit:

$$\begin{aligned} \nabla_{\mathbf{k}} \sigma(\hat{\mathbf{k}}) &= \hat{\tau} \{\hat{\mathbf{k}}' \cdot \nabla_{\mathbf{k}} \boldsymbol{\eta}(\hat{\mathbf{k}}) + \boldsymbol{\eta}(\hat{\mathbf{k}})' \cdot \nabla_{\mathbf{k}} \mathbf{F}(\hat{\mathbf{q}}, \hat{\mathbf{k}})\} - \hat{\mu} \nabla_{\mathbf{k}} cs(\hat{\mathbf{k}}) \\ &= -\hat{\tau} \{\boldsymbol{\psi}(\hat{\mathbf{k}})' - \boldsymbol{\varphi}(\hat{\mathbf{k}})'\} - \hat{\mu} \nabla_{\mathbf{k}} cs(\hat{\mathbf{k}}). \end{aligned} \quad (29)$$

Note that  $\nabla_{\mathbf{k}} v(\mathbf{k}) = \boldsymbol{\eta}(\mathbf{k})' - \boldsymbol{\varphi}(\mathbf{k})'$  holds from the envelope theorem. Furthermore, since  $\nabla_{\mathbf{k}} v(\mathbf{k}) = \nabla_{\mathbf{k}} cs(\mathbf{k}) + \nabla_{\mathbf{k}} \rho(\mathbf{k})$  and  $\nabla_{\mathbf{k}} \rho(\mathbf{k}) = \boldsymbol{\eta}(\mathbf{k})' - \boldsymbol{\psi}(\mathbf{k})'$ , we have the following equality:

$$\nabla_{\mathbf{k}} cs(\mathbf{k}) = \boldsymbol{\psi}(\mathbf{k})' - \boldsymbol{\varphi}(\mathbf{k})'. \quad (30)$$

Hence, we obtain the following condition in the limit:

$$\nabla_{\mathbf{k}} \sigma(\hat{\mathbf{k}}) = -(\hat{\tau} + \hat{\mu}) \nabla_{\mathbf{k}} cs(\hat{\mathbf{k}}). \quad (31)$$

Therefore, the three surfaces  $\{\mathbf{k} \mid \sigma(\mathbf{k}) = \sigma(\hat{\mathbf{k}})\}$ ,  $\{\mathbf{k} \mid u(\mathbf{k}) = u(\hat{\mathbf{k}})\}$ , and the regulatory constraint of the extended price cap mechanism are tangent to each other at  $\hat{\mathbf{k}} = \mathbf{k}^*$ . This completes the proof. ■

### Proof of Proposition 3

Using similar notations as in the proof of Proposition 2, the first inequality of the extended V-F mechanism can be expressed as  $\delta(\mathbf{k}) \leq c(\bar{\mathbf{k}}) + \rho(\bar{\mathbf{k}})$ .

Suppose that  $\sigma(\bar{\mathbf{k}}) = \boldsymbol{\eta}(\bar{\mathbf{k}})' \cdot \bar{\mathbf{k}} - c(\bar{\mathbf{k}}) \geq 0$  in the previous period  $t-1$ . Since we assume that  $\boldsymbol{\eta}(\alpha \mathbf{k})$  approaches 0 as  $\alpha > 1$  gets large, there exists  $\alpha$  such that

$\boldsymbol{\eta}(\alpha\bar{\mathbf{k}})' \cdot \bar{\mathbf{k}} - c(\bar{\mathbf{k}}) = 0$ . Moreover,  $\mathbf{F}(\bar{\mathbf{q}}, \alpha\bar{\mathbf{k}}) = \mathbf{F}(\bar{\mathbf{q}}, \bar{\mathbf{k}})$  holds for such  $\alpha$  from the assumption that  $\mathbf{F}(\bar{\mathbf{q}}, \mathbf{k})$  is homogenous of degree 0 with respect to  $\mathbf{k}$ . Thus, it follows that such  $\mathbf{k} = \alpha\bar{\mathbf{k}}$  satisfies the first inequality of the mechanism:

$$\begin{aligned}\delta(\alpha\bar{\mathbf{k}}) &= \boldsymbol{\eta}(\alpha\bar{\mathbf{k}})' \cdot \bar{\mathbf{k}} + \boldsymbol{\eta}(\bar{\mathbf{k}})' \cdot \mathbf{F}(\bar{\mathbf{q}}, \alpha\bar{\mathbf{k}}) \\ &= \boldsymbol{\eta}(\alpha\bar{\mathbf{k}})' \cdot \bar{\mathbf{k}} + \boldsymbol{\eta}(\bar{\mathbf{k}})' \cdot \mathbf{F}(\bar{\mathbf{q}}, \bar{\mathbf{k}}) \\ &= c(\bar{\mathbf{k}}) + \rho(\bar{\mathbf{k}}).\end{aligned}\tag{32}$$

Since, by assumption,  $cs(\alpha\bar{\mathbf{k}}) \geq cs(\bar{\mathbf{k}})$  holds for such  $\alpha$ ,  $\mathbf{k} = \alpha\bar{\mathbf{k}}$  also satisfies the second inequality of the mechanism. Furthermore,  $c(\alpha\bar{\mathbf{k}}) \leq \alpha c(\bar{\mathbf{k}})$  follows from the assumption that  $c(\mathbf{k})$  exhibits no decreasing return to scale. Thus, we have

$$\begin{aligned}\sigma(\mathbf{k}) &\geq \sigma(\alpha\bar{\mathbf{k}}) \\ &= \boldsymbol{\eta}(\alpha\bar{\mathbf{k}})' \cdot \alpha\bar{\mathbf{k}} - c(\alpha\bar{\mathbf{k}}) \\ &\geq \boldsymbol{\eta}(\alpha\bar{\mathbf{k}})' \cdot \alpha\bar{\mathbf{k}} - \alpha c(\bar{\mathbf{k}}) \\ &= \alpha \{ \boldsymbol{\eta}(\alpha\bar{\mathbf{k}})' \cdot \bar{\mathbf{k}} - c(\bar{\mathbf{k}}) \} \\ &= 0\end{aligned}\tag{33}$$

for period  $t$ . Assuming that  $\sigma(\mathbf{k}^0) \geq 0$  in the first period, and by induction,  $\sigma(\mathbf{k}) \geq 0$  holds for all  $t$ . In other words, the extended V-F mechanism is feasible for the Transco in the sense that it can make a non-negative profit in all periods.

Since  $cs(\mathbf{k}) \geq cs(\bar{\mathbf{k}})$  holds under the extended V-F mechanism,  $\{cs(\mathbf{k})\}_{t=0}^{\infty}$  is a bounded non-decreasing sequence, and hence the limit  $\lim_{t \rightarrow \infty} cs(\mathbf{k}) = cs(\tilde{\mathbf{k}})$  exists. At the limit point  $\tilde{\mathbf{k}}$ ,  $\delta(\tilde{\mathbf{k}}) \leq c(\tilde{\mathbf{k}}) + \rho(\tilde{\mathbf{k}})$  yields  $\sigma(\tilde{\mathbf{k}}) \leq 0$ . On the other hand, since  $\sigma(\mathbf{k}) \geq 0$  holds for all  $t$ , it follows that  $\sigma(\tilde{\mathbf{k}}) = 0$ ; that is, the Transco's profit will vanish in the limit. Note that  $\{\sigma(\mathbf{k})\}_{t=0}^{\infty}$  is not a non-decreasing sequence, although  $\sigma(\mathbf{k})$  is non-negative.

Considering that the Transco's profit is zero in the limit, we can restate the regulator's problem as  $\max_{\mathbf{k}} \{u(\mathbf{k}) \equiv cs(\mathbf{k}) : \sigma(\mathbf{k}) = 0\}$ . The first-order condition can be rewritten as follows:

$$\nabla_{\mathbf{k}} cs(\mathbf{k}) = -\tilde{\zeta} \nabla_{\mathbf{k}} \sigma(\mathbf{k}),\tag{34}$$

On the other hand, the Transco's problem in each period can be expressed as  $\max_{\mathbf{k}} \{ \sigma(\mathbf{k}) : \delta(\mathbf{k}) \leq c(\bar{\mathbf{k}}) + \rho(\bar{\mathbf{k}}), cs(\mathbf{k}) \geq cs(\bar{\mathbf{k}}) \}$ . By the same argument in the proof of Proposition 2, we obtain the following condition in the limit:

$$\nabla_{\mathbf{k}} \sigma(\tilde{\mathbf{k}}) = -(\tilde{\tau} + \tilde{\mu}) \nabla_{\mathbf{k}} cs(\tilde{\mathbf{k}}).\tag{35}$$

Therefore, the three surfaces  $\{\mathbf{k} \mid \sigma(\mathbf{k}) = 0\}$ ,  $\{\mathbf{k} \mid u(\mathbf{k}) = u(\tilde{\mathbf{k}})\}$ , and the regulatory

constraint of the extended V-F mechanism are tangent to each other at  $\tilde{\mathbf{k}} = \mathbf{k}^*$ . This completes the proof. ■

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