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Abstract

The sources of economic fluctuations discussed in the existing literature are information asymmetry, incomplete contracts, and serially correlated exogenous shocks. We show that an economy may fluctuate cyclically without these assumptions if production of payment services (deposit money) necessitates physical capital.

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In the existing literature on economic fluctuations and financial instability, the sources of fluctuations have been envisioned as information asymmetry or incomplete contracts between financier and financee (see Bernanke and Gertler [1989] and Kiyotaki and Moore [1997]), or exogenous shocks on productivity or preference that are serially correlated (Cooley [1995]). We show that without these assumptions business cycles may occur in a neoclassical growth model, which is fairly standard except for the existence of two different kinds of capital in the production and financial sectors.

1 Model

The economy is a continuous time neoclassical growth model, in which there are consumers, firms, banks, and a government. At each point in time, four technology disturbances z_{it} $(i = 1, 2, 3, 4)$ hit the economy (see below). For analytical simplicity, we assume that z_{it}'s are deterministic functions of t and that all agents know their exact paths. The production function for consumer goods y_t is $y_t = e^{z_{1t}} A k_t^{\alpha} n_t^{1-\alpha}$, where k_t is the capital for producing consumer goods and n_t is the labor input. Here, k_t evolves by

$$\dot{k}_t = i_t - \delta_f e^{z_{2t}} k_t, \tag{1}$$

where i_t is the investment in the production sector. At each point in time a firm chooses k_t and n_t to maximize $\pi_t^f = e^{z_{1t}} A k_t^{\alpha} n_t^{1-\alpha} - r_t^k k_t - w_t n_t$, where r_t^k is the rental rate for capital k and w_t is the wage rate. The production function for deposit money (i.e., payment services) d_t is $d_t = e^{z_{3t}} B h_t^{\beta} m_t^{1-\beta}$, where h_t is the capital for producing deposits and m_t is (real) cash reserves, i.e., the stock of cash.[1] Here, h_t and m_t evolve by

$$\dot{h}_t = j_t - \delta_b e^{z_{4t}} h_t, \tag{2}$$

$$\dot{m}_t = x_t, \tag{3}$$

where j_t is the investment in the banking sector, and x_t is cash injection to the consumer by the government. At each point in time a bank chooses h_t and m_t to maximize

[1] Chari, Christiano, and Eichenbaum (1995) assume similar technology for producing deposit money. But in their model, the same capital can be used for producing goods or financial services.

$\pi_t^b = (r_t^L - r_t^d)e^{z_{3t}}Bh_t^\beta m_t^{1-\beta} - r_t^d m_t - r_t^h h_t$, where r_t^L is the loan rate, r_t^d is the deposit rate, and r_t^h is the rental rate for capital h. The right-hand side of π_t^b means that when a bank provides a loan to a consumer, it places bank deposits in the consumer's account while the consumer makes a promise to repay them at the loan rate r_t^L. Note that the deposit d_t yields the interest $r^d d_t$. We assume the following deposit-in-advance constraint: The consumer must hold bank deposits in advance to buy a part of her consumption and investment $(\eta c_t + \psi i_t)$, where $0 < \eta < 1$ and $0 < \psi < 1$. Therefore, she needs to borrow bank deposits d_t from a bank.

The consumer is endowed with 1 unit of labor at each point in time, and solves $\max_{c_t,n_t,i_t,j_t,d_t,s_t} \int_0^\infty u(c_t)e^{-\rho t}dt$ subject to $c_t + i_t + j_t + s_t + (1+r_t^L)d_t \leq x_t + w_t n_t + r_t^d m_t + r_t^k k_t + r_t^h h_t + (1+r_t^d)d_t + \pi_t^f + \pi_t^b$, $\eta c_t + \psi i_t \leq d_t$, $0 \leq n_t \leq 1$, (1), (2), and (3), where $u(c) = \frac{c^{1-\theta}}{1-\theta}$ $(\theta > 0)$ and s_t is an addition in cash holdings. From the first order conditions for the firm's and the bank's problems and the equilibrium conditions ($n_t = 1$ and $s_t = x_t$), we can obtain the following reduced form: $\max_{c_t,i_t,j_t} \int_0^\infty u(c_t)e^{-\rho t}dt$ subject to $c_t + i_t + j_t \leq e^{z_{1t}}Ak_t^\alpha$, $\chi c_t + i_t \leq e^{z_{3t}}B' h_t^\beta m_t^{1-\beta}$, (1) and (2), where $\chi = \eta/\psi$ and $B' = B/\psi$. The Hamiltonian for this problem is $\mathcal{H} = e^{-\rho t}[u(c_t) + \lambda_t(e^{z_{1t}}Ak_t^\alpha - e^{z_{3t}}B_t h_t^\beta - (1-\chi)c_t - e^{z_{4t}}\delta_b h_t) + q_t(e^{z_{3t}}B_t h_t^\beta - \chi c_t - e^{z_{2t}}\delta_f k_t)]$, where $B_t \equiv B' m_t^{1-\beta}$, and λ_t and q_t are the Lagrange multipliers. The dynamics of the economy are described as the paths of four variables: $\{k_t, h_t, \lambda_t, q_t\}$. We assume the following:

Assumption *The government determines x_t such that $\forall t$ $|B_t - B^*| < \epsilon$ where ϵ is a small number, and $\lim_{T\to\infty} \frac{1}{T}\int_0^T (B_t - B^*)dt = 0$. The exogenous shocks z_{it} $(i = 1,2,3,4)$ also satisfy $\forall t$ $|z_{it}| < \epsilon$, and $\lim_{T\to\infty} \frac{1}{T}\int_0^T z_{it}dt = 0$.*

The steady state $(k^*, h^*, \lambda^*, q^*)$ that corresponds to $B_t = B^*$ and $z_{it} = 0$ is determined by $k^* \equiv k(h^*) = \left[\frac{\rho+\delta_f}{\alpha A} + \frac{(\rho+\delta_f)(\rho+\delta_b)}{\alpha\beta AB^*}h^{*1-\beta}\right]^{-\frac{1}{1-\alpha}}$, $\frac{q^*}{\lambda^*} = (\rho+\delta_f)^{-1}\alpha Ak^{*\alpha-1}$, $\{(1-\chi)\lambda^* + \chi q^*\}^{-\frac{1}{\theta}} = \frac{1}{\chi}\{B^* h^{*\beta} - \delta_f k^*\}$, and $F(h^*) = 0$, where $F(h) = B^* h^\beta - \delta_f k(h) + \frac{\chi}{\chi-1}(Ak(h)^\alpha - B^* h^\beta - \delta_b h)$. It is easily shown that the steady state is unique if $0 < \chi \leq 1$. For $\chi > 1$, the steady state is also unique, since $F(0) > 0 > \lim_{h\to\infty} F(h)$ and $\forall h(> 0)$ $F'(h) < 0$.

We focus our analysis on the dynamics in the neighborhood of the steady state. Following Barro and Sala-I-Martin (1995), we can log-linearize the dynamics of this system.

We define that $\boldsymbol{x_t} = (\ln \frac{h_t}{h^*}, \ln \frac{k_t}{k^*}, \ln \frac{\lambda_t}{\lambda^*}, \ln \frac{q_t}{q^*})'$, $\boldsymbol{b} = (-B^* h^{*\beta-1}, B^* h^{*\beta} k^{*-1}, -(\rho+\delta_b), 0)'$,

$$A = \begin{pmatrix} -\beta B^* h^{*\beta-1} - \delta_b & \alpha A \frac{k^{*\alpha}}{h^*} & \frac{1-\chi}{\theta}\frac{(\rho+\delta_f)E}{D} & \frac{\chi}{\theta}\frac{\alpha A k^{*\alpha-1}E}{D} \\ \beta B^* \frac{h^{*\beta}}{k^*} & -\delta_f & \frac{1-\chi}{\theta}\frac{(\rho+\delta_f)F}{D} & \frac{\chi}{\theta}\frac{\alpha A k^{*\alpha-1}F}{D} \\ (1-\beta)(\rho+\delta_b) & 0 & \rho+\delta_b+\beta B^* h^{*\beta-1} & -(\rho+\delta_b+\beta B^* h^{*\beta-1}) \\ 0 & (1-\alpha)(\rho+\delta_f) & -(\rho+\delta_f) & (\rho+\delta_f) \end{pmatrix},$$

where $D = (1-\chi)(\rho+\delta_f)+\chi\alpha A k^{*\alpha-1}$, $E = A\frac{k^{*\alpha}}{h^*} - B^* h^{*\beta-1} - \delta_b$, and $F = B^*\frac{h^{*\beta}}{k^*} - \delta_f$; and $\boldsymbol{\omega_t} = (\omega_{1t}, \omega_{2t}, \omega_{3t}, \omega_{4t})'$, where $\omega_{1t} = A\frac{k^{*\alpha}}{h^*}z_{1t} - B^* h^{*\beta-1} z_{3t} - \delta_b z_{4t}$, $\omega_{2t} = B^*\frac{h^{*\beta}}{k^*}z_{3t} - \delta_f z_{2t}$, $\omega_{3t} = \delta_b z_{4t} + \beta B^* h^{*\beta-1} z_{3t} - \frac{q^*}{\lambda^*}\beta B^* h^{*\beta-1} z_{3t}$, and $\omega_{4t} = \delta_f z_{2t} - \frac{\lambda^*}{q^*}\alpha A k^{*\alpha-1} z_{1t}$. Note that $\forall t\ |\omega_{it}| < \epsilon'$ for a small number ϵ' and $\lim_{T\to\infty}\frac{1}{T}\int_0^T \omega_{it}dt = 0$. The log-linearized dynamics around the steady state are described as $\dot{\boldsymbol{x}}_{\boldsymbol{t}} = A\boldsymbol{x_t} + \boldsymbol{b}\ln\frac{B_t}{B^*} + \boldsymbol{\omega_t}$.

As a benchmark, let us examine the case where $\boldsymbol{\omega_t} = \boldsymbol{0}$ and $B_t = B^*$ for all t. In this case, the dynamics are described as $\dot{\boldsymbol{x}}_{\boldsymbol{t}} = A\boldsymbol{x_t}$, the general solution to which is $\boldsymbol{x_t^g} = \sum_{i=1}^4 C_i e^{\mu_i t}\boldsymbol{x_i}$, where $\boldsymbol{x_i}$ ($|\boldsymbol{x_i}| = 1$) is the eigenvector corresponding to the eigenvalue μ_i ($i = 1,2,3,4$) of matrix A, and C_i is the constant determined by the initial and the transversality conditions (see Hirsch and Smale [1974]). We confirmed numerically that for a wide range of parameter values two eigenvalues are positive real numbers or complex numbers with positive real parts and the other two are either negative real numbers (see, for example, the white region of the figure) or complex numbers that are mutually conjugate with real parts that are negative (the shaded region of the figure). Suppose that parameter values are in the shaded region. We can denote that $\mu_1 = -r+i\phi$ and $\mu_2 = -r-i\phi$, $\mu_3 > 0$, and $\mu_4 > 0$, where $r > 0$ and $\phi > 0$. The transversality conditions ($\lim_{t\to\infty}\lambda_t k_t = \lim_{t\to\infty} q_t h_t = 0$) imply $C_3 = C_4 = 0$. Thus, the solution is $\boldsymbol{x_t} = \boldsymbol{v_1}e^{-rt}\cos\phi t + \boldsymbol{v_2}e^{-rt}\sin\phi t$, where $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are real valued vectors that are linear combinations of $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$. Therefore, in the neighborhood of the steady state, the economy cyclically fluctuates at the frequency ϕ and asymptotically converges to the steady state. This behavior is called a damped oscillation at frequency ϕ.

In the case where $\boldsymbol{\omega_t} \neq \boldsymbol{0}$ and $\forall t\ B_t = B^*$, the system exhibits a forced oscillation. The economy continues to fluctuate, and the frequency corresponding to the largest power spectrum of the fluctuation is also ϕ if $\boldsymbol{\omega_t}$ is approximated as an i.i.d. random shock. In the case where $\boldsymbol{\omega_t} = \mathbf{0}$ and $B_t \neq B^*$, the solution is $\boldsymbol{x_t} + \sum_{i=1}^{4} b_i \boldsymbol{x_i} e^{\mu_i t} \int_0^t e^{-\mu_i s} \ln \frac{B_s}{B^*} ds$, where b_i is the inner product of $\boldsymbol{b}$ and $\boldsymbol{x_i}$. In this case too, it can be said that the economy fluctuates mainly at the frequency ϕ if B_t is approximated as an i.i.d. shock.

2 Concluding Remarks

The figure implies that oscillation occurs when B' is large and β is small. This means that the economy may become unstable as the financial sector becomes more productive (a large B') and physical capital becomes less important in producing financial services (a small β). Thus this model implies that financial innovation may make the economy unstable. Note that ϕ (the main frequency of fluctuations) is a structural parameter, which is independent from the nature of exogenous shocks z_{it}. This implies that the frequencies of business cycles may not be determined by exogenous temporary shocks on technology or preference, but by the fundamental structure of the economic system.

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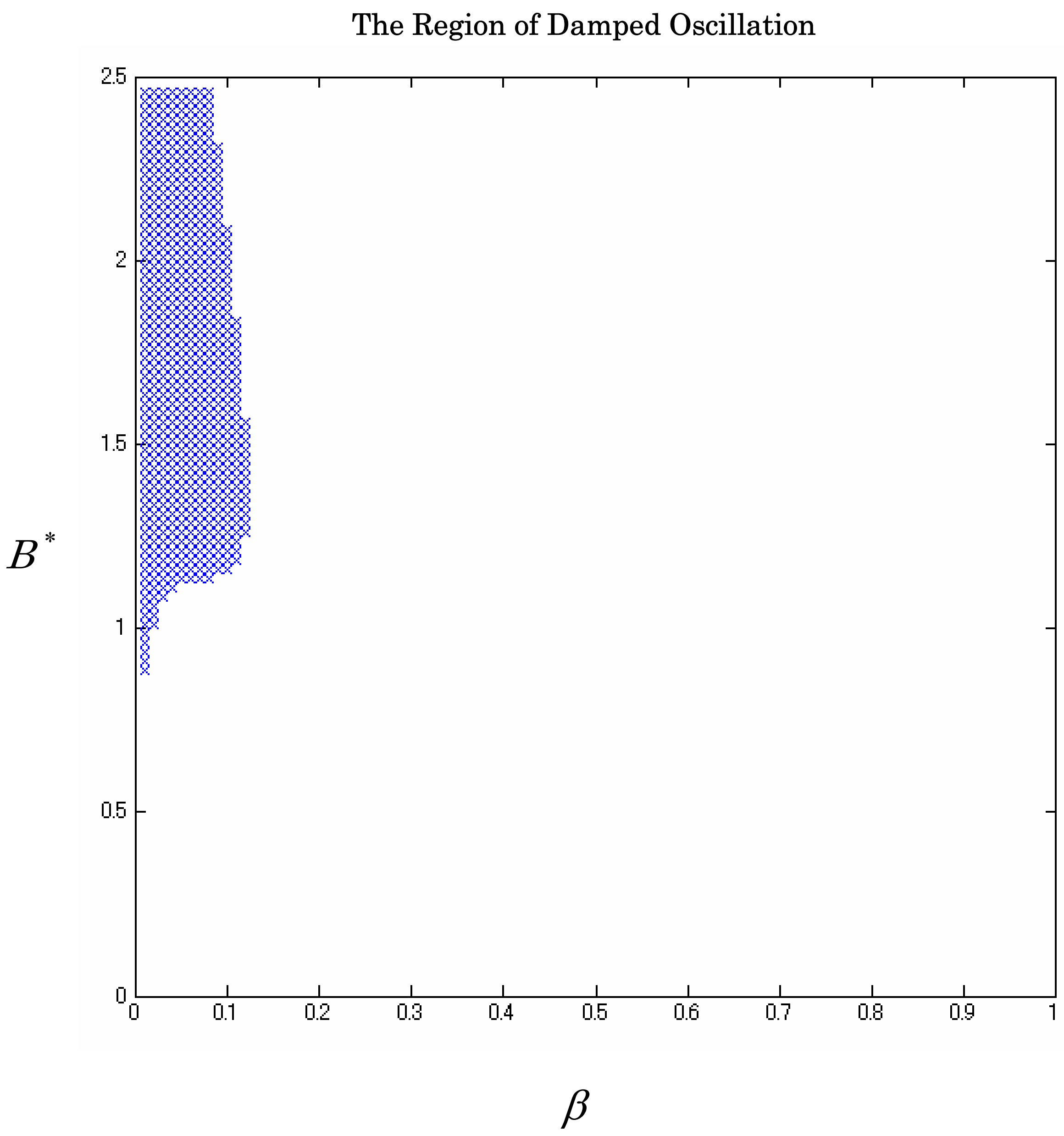

Parameters: $A = 1$

$\alpha = 0.35$

$\rho, \delta_b, \delta_f = 0.002$

$\theta = 1.5$

$\chi = 0.5$