



# **FNet 2013 Kyoto**

## **A Model of Macroprudential Policy**

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**Are  
Economic Behavior  
and  
Motion of “Particle”  
Fundamentally Different?**



**Most Economists  
Think  
They are  
Fundamentally Different.**



**Thus, Economics must be  
based on  
Microeconomic Analysis  
of Purposeful  
Economic Behavior**



**This Leads Economists  
to the Analysis  
based on  
the Representative Agent.**



**Every student of economics knows that behavior of dynamically optimizing economic agent such as the Ramsey consumer is described by the Euler equation for a problem of calculus of variation. On the surface, such a sophisticated economic behavior must look remote from “mechanical” movements of an inorganic particle which only satisfy the law of motion. However, every student of physics knows that the Newtonian law of motion is actually nothing but the Euler equation for a certain variational problem. It is called the principle of least action: see Chapter 19 of Feynmann (1964)’s *Lectures on Physics*, Vol. II. Therefore, behavior of dynamically optimizing economic agent and motions of inorganic particle are on a par to the extent that they both satisfy the Euler equations for respective variational problems. The method of statistical physics can be usefully applied not because motions of micro units are “mechanical,” but because object under investigation comprises many micro units individual movements of which we are unable to know.**



JAPAN-US CENTER UFB BANK MONOGRAPHS ON INTERNATIONAL FINANCIAL MARKETS  
Praise for *Reconstructing Macroeconomics*

"Thoughtful macroeconomists are uncomfortably aware that consumers, firms, and workers vary widely in their local environments, perceptions, and beliefs. Ignoring this heterogeneity, as 'modern macro' does, is a likely source of systematic error. Aoki and Yoshikawa propose to repair this failure by modeling the macroeconomy explicitly as a cloud of interacting particles. The goal is to deduce the distributions of economic characteristics that describe the system as a whole. This puts more emphasis on statistical properties and less on the internal decision making of each agent. There are already some surprising beginning results, including a novel treatment of aggregate demand, and one can expect more when their approach is combined with standard economic reasoning. This is the start, not the finish, of a potentially far-reaching research program. It should excite the curiosity of all those thoughtful macroeconomists."

— Robert M. Solow, *Nobel Laureate, Massachusetts Institute of Technology*

"This book is a bold and daring challenge to the growing influence of neoclassical equilibrium theory in the field of modern macroeconomics. Not simply an approach to traditional Keynesian theory that attempts to refine it and make it more accurate, the treatment makes use of a new methodology in statistical physics and combinatorial stochastic processes to mount a direct challenge to real business cycle theory and rational expectations theory. This technique makes it possible to analyze the interactions of a large number of fluctuating micro agents. Professor Aoki has made important contributions to the application of statistical physics to economics, and Professor Yoshikawa is a leading Japanese economist who has done outstanding work in the fields of both theoretical and empirical economics. This book is the superb product of the optimum combination of these two scholars' different talents."

— Ryoan Sato, *New York University and University of Tokyo*

"Masanao Aoki and Hiroshi Yoshikawa have written no less than the foundation of a new approach (and I believe the right one) to the core problem of macroeconomics, which is to aggregate behaviors by stressing the importance of the heterogeneity and variability of real economic agents. Getting inspiration from and adapting the concepts and tools of statistical physics, they masterfully derive important and novel insights on the most crucial open problems of the field: the principle of effective demand, role of uncertainty, sticky prices/wages, and the endogenous business cycle. By systematically discussing and comparing their theory with empirical data and real economic situations, this book is perhaps the first successful effort to develop macroeconomics as a real science on par with physics, with falsifiable hypotheses underpinned by sound micro-principles and testable predictions."

— Didier Sornette, *Swiss Federal Institute of Technology, Zurich*

"This book shows the impossibility of efficient equilibria in economies with market clearing mainstream hypotheses when such an economy is populated by a large number of heterogeneous agents. In such a case, Aoki and Yoshikawa show that, through combinatorial stochastic processes, a new approach to macroeconomics is not only possible it is real and this book shows how to reach it."

— Mauro Gallegati, *Università Politecnica delle Marche*

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Aoki Yoshikawa

Reconstructing Macroeconomics

CAMBRIDGE

RECONSTRUCTING MACROECONOMICS  
A PERSPECTIVE FROM STATISTICAL PHYSICS AND  
COMBINATORIAL STOCHASTIC PROCESSES



MASANAO AOKI AND HIROSHI YOSHIKAWA

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# Solow's comment

**“Thoughtful macroeconomists are uncomfortably aware that consumers, firms, and workers vary widely in their local environments, perceptions, and beliefs. Ignoring this heterogeneity, as ‘modern macro’ does, is a likely source of systematic error. Aoki and Yoshikawa propose to repair this failure by modeling the macroeconomy explicitly as a cloud of interacting particles. The goal is to deduce the distributions of economic characteristics that describe the system as a whole. This puts more emphasis on statistical properties and less on the internal decision making of each agent. There are already some surprising beginning results, including a novel treatment of aggregate demand, and one can expect more when their approach is combined with standard economic reasoning. This is the start, not the finish, of a potentially far-reaching research program. It should excite the curiosity of all those thoughtful macroeconomists.” Robert M. Solow (2007)**



# Model

$$x = \frac{k}{K} \quad (k = 1, \dots, K) \quad (1)$$

$$r = K(1 - x)\eta_1(x), \quad (2)$$

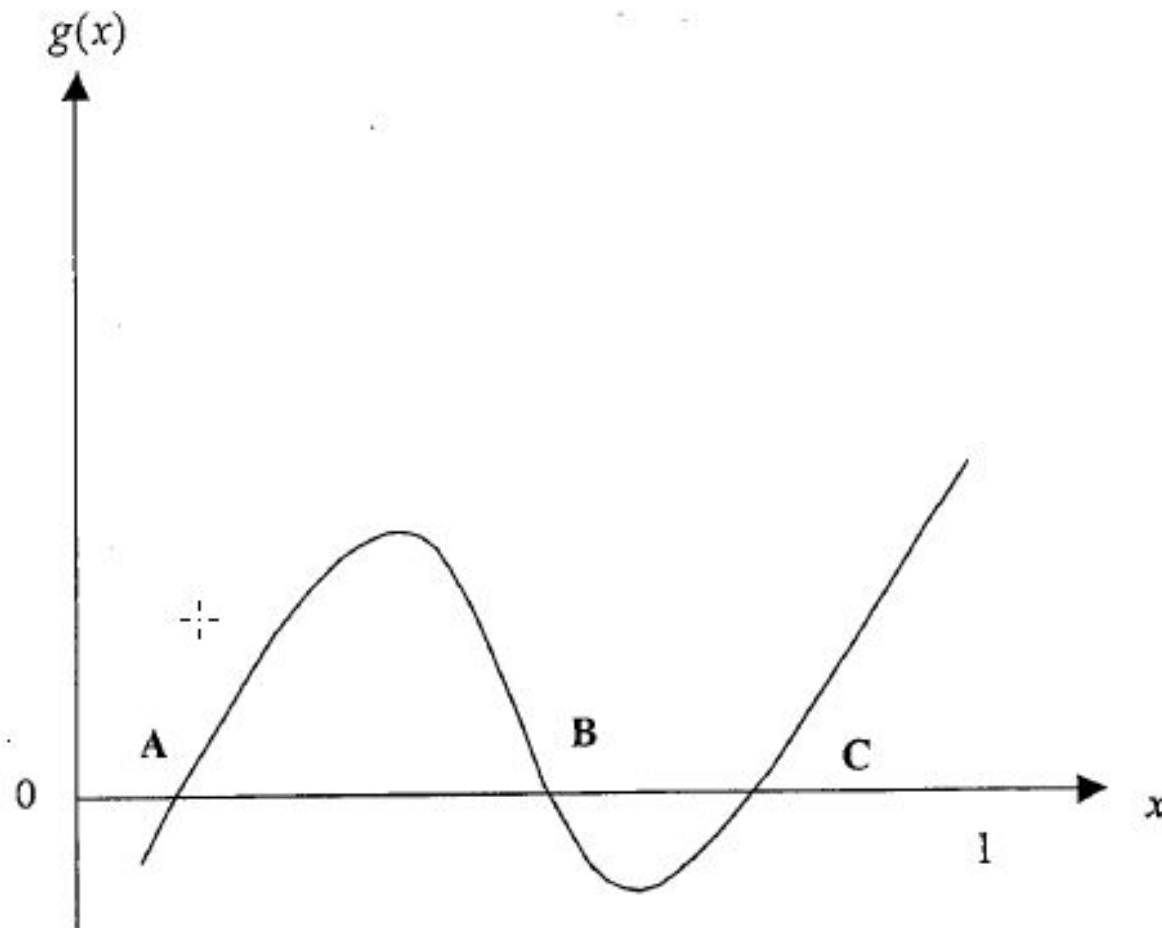
$$l = Kx\eta_2(x). \quad (3)$$

$$\eta_1(x) = X^{-1}e^{\beta g(x)} \quad (\beta > 0) \quad (4)$$

$$\eta_2(x) = 1 - \eta_1(x) = X^{-1}e^{-\beta g(x)} \quad (5)$$

$$X = e^{\beta g(x)} + e^{-\beta g(x)} \quad (6)$$

# Figure 1 : $g(x)$ Function Representing Bank Behavior



# Model

$$\begin{aligned}\frac{\partial P(k, t)}{\partial t} &= K \left( x + \frac{1}{K} \right) \eta_2 \left( x + \frac{1}{K} \right) P(k + 1, t) \\ &+ K \left( 1 - x + \frac{1}{K} \right) \eta_1 \left( x - \frac{1}{K} \right) P(k - 1, t) \\ &- [Kx\eta_2(x) + K(1 - x)\eta_1(x)] P(k, t).\end{aligned}\quad (7)$$

$$\frac{d\bar{x}_t}{dt} = \dot{\bar{x}}_t = \bar{x}_t \eta_2(\bar{x}_t) - (1 - \bar{x}_t) \eta_1(\bar{x}_t)\quad (8)$$

$$\frac{\eta_1(\bar{x})}{\eta_2(\bar{x})} = \frac{\bar{x}}{1 - \bar{x}}.\quad (9)$$

# Model

$$2\beta g(\bar{x}) = \log\left(\frac{\bar{x}}{1-\bar{x}}\right). \quad (10)$$

$$g(\bar{x}) = 0. \quad (11)$$

$$U(x) = -2 \int^x g(y) dy - \frac{1}{\beta} H(x). \quad (12)$$

$$H(x) = -x \ln x - (1-x) \ln(1-x). \quad (13)$$

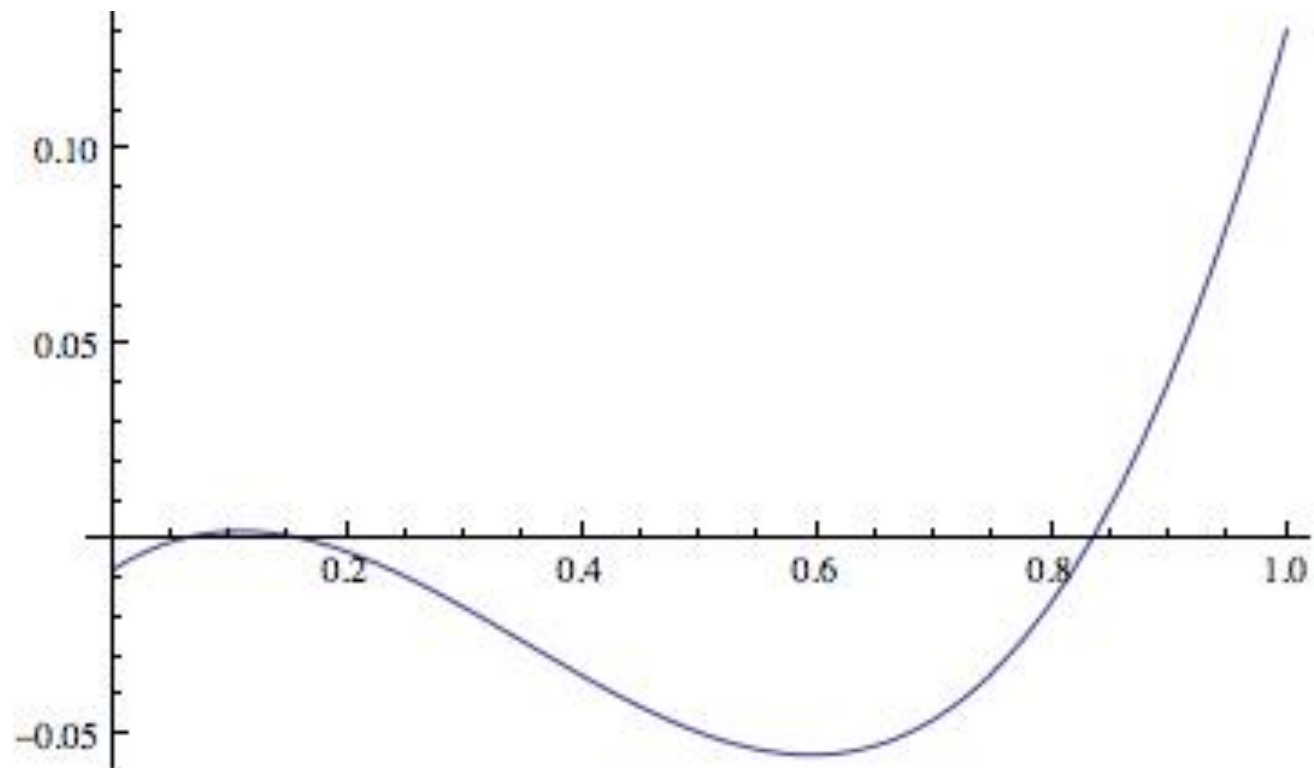
# Model

$$\begin{aligned}\log_K C_k &= \log\left(\frac{K!}{(K-k)!k!}\right) \\ &= K\left[-\left(\frac{k}{K}\right)\log\left(\frac{k}{K}\right) - \left(1 - \frac{k}{K}\right)\log\left(1 - \frac{k}{K}\right)\right] \\ &= KH(x)\end{aligned}\tag{14}$$

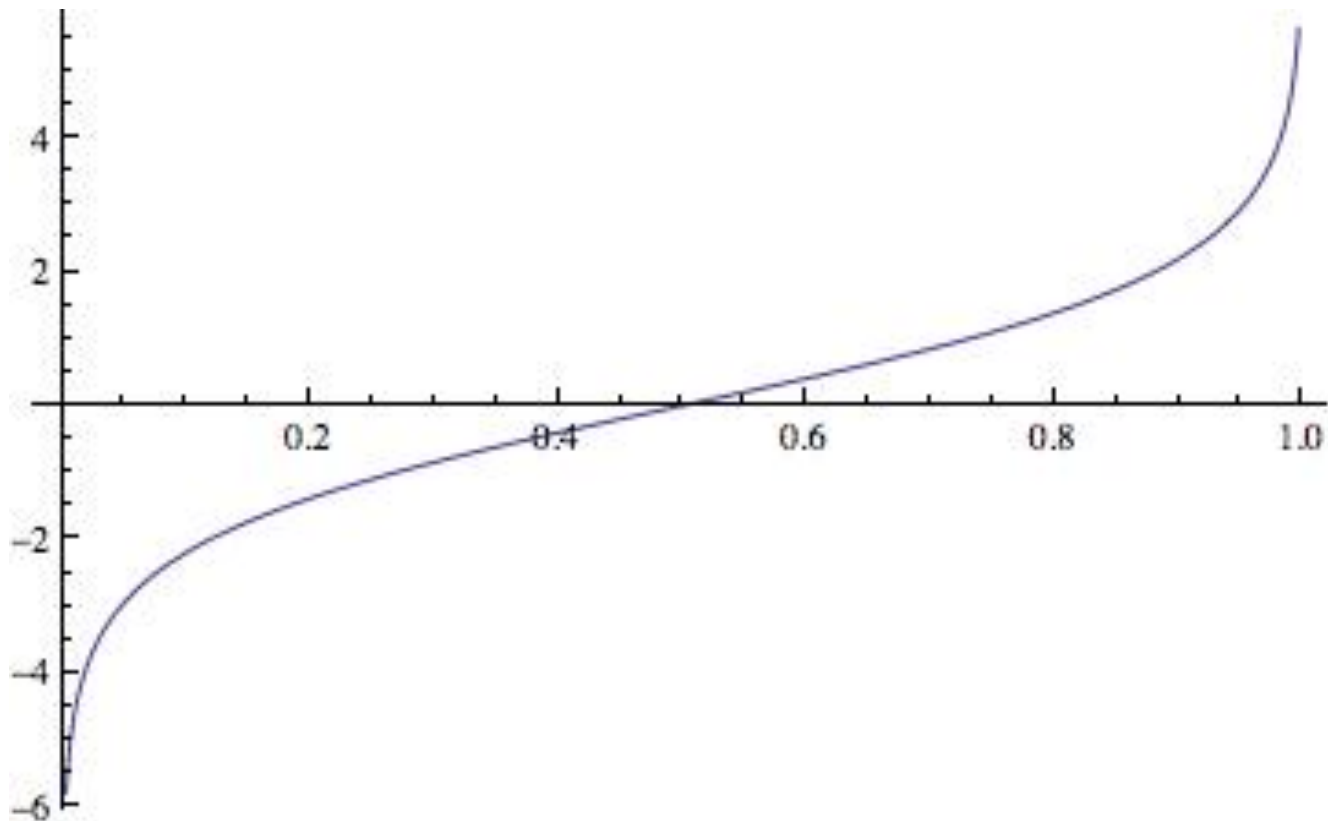
$$\frac{dH(x)}{dx} = \log\left(\frac{1-x}{x}\right)\tag{15}$$

$$U'(\bar{x}) = -2g(\bar{x}) - \frac{1}{\beta}H'(\bar{x}) = -2g(\bar{x}) + \frac{1}{\beta}\log\left(\frac{\bar{x}}{1-\bar{x}}\right) = 0\tag{16}$$

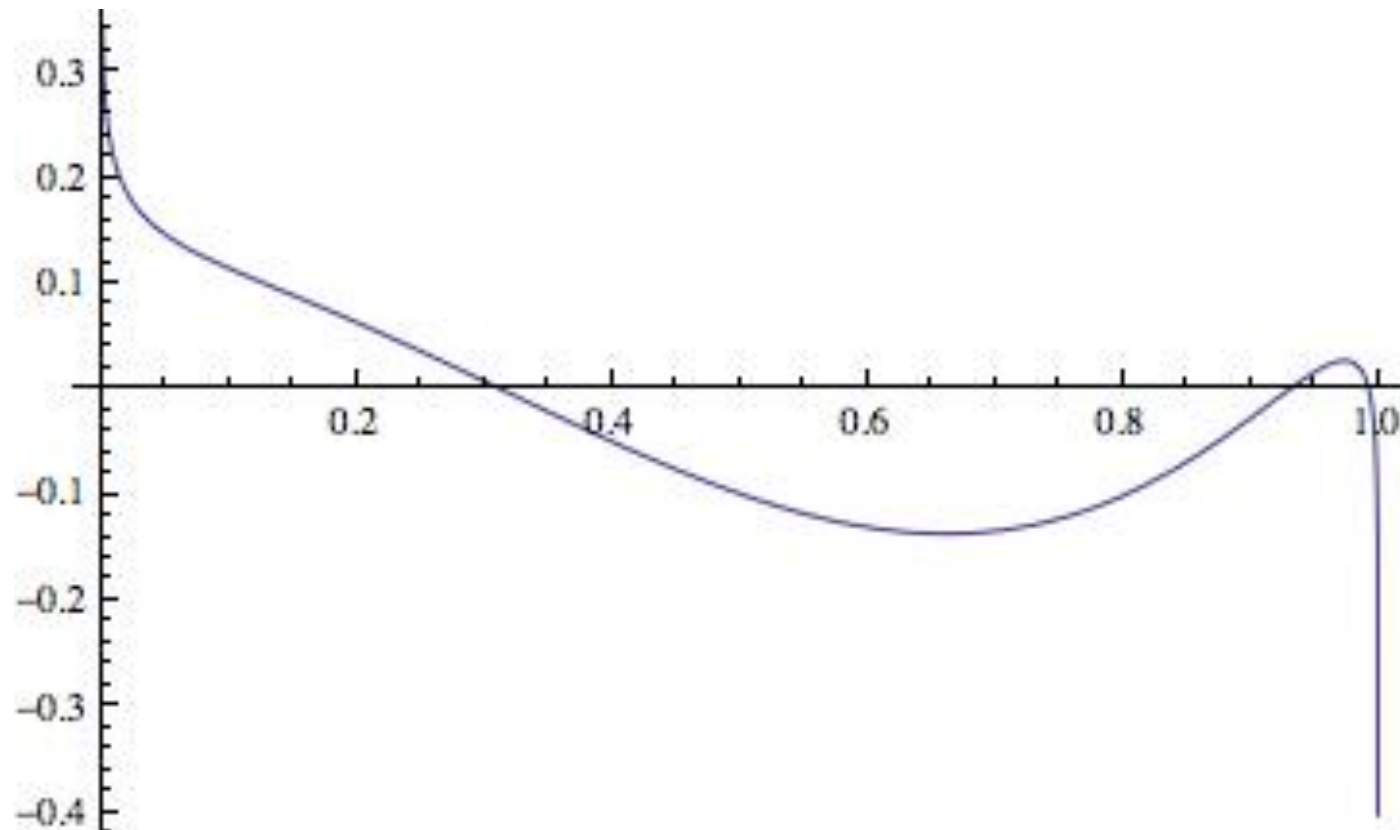
## Figure 2 : A Example of $g(x)$ Function



## Figure 3 : Entropy Term $\log(x/(1-x))$



## Figure 4 : $U'(x)$ or Equation (16)





## Figure 5 : Multiple Equilibrium

