Lagos-Wrightの枠組みを基にした銀行危機の貨幣的モデル

小林慶一郎

独立行政法人経済産業研究所(RIETI)

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動　機

- **現在の金融危機に対する分析の枠組み**
  - 一般的な景気循環（DSGE）モデルでは金融危機を分析できない
  - DSGEモデルと両立し得る銀行モデルが必要（貨幣と財貨の扱いやすい区別が必要）
  - 現在の政策の有効性を評価する統一された枠組み（財政出動、金融緩和、銀行改革）
- **銀行業務と金融危機の本質:**
  - 流動性保険（Bryant, Diamond-Dybvig, Allen-Gale）
  - ホールドアップ問題に対する最善の契約（Diamond-Rajan）
  - 支払いサービスか、交換手段の提供か（Lagos-Wright を基にした Berentsen-Camera-Waller）

発表の要旨 (1/2)

金融危機の扱いやすい貨幣的モデルを構築する

- 銀行サービス＝支払仲介
  - 現金経済 – 現金の流通　一度のみ:
    ★Buyers（買い手） ⇒ Sellers（売り手）
  - 銀行経済 – 現金の流通　$1/\rho = J$回
    ★Bank ⇒ Buyers j ⇒ Sellers j ⇒ Bank ⇒ Buyers $j+1$ ⇒ ・・・ (J times)
  - 銀行危機 — 全ての売り手が預金をやめる:
    ★銀行破たん（外因性; 内因性）を予想
    ★Bank ⇒ Buyers 1 ⇒ Sellers 1 ⇒ X
    ★第一段階として、銀行は現金準備不足に陥る
    ★Buyers 1は預金を引き出し、財貨を購入する
    ★残りのBuyer (Buyers 2, ・・・ , Buyers J) は財貨を購入できない。
    （預金者へのSequential Service Constraintによる問題）
    ★財貨の需要が急激に落ち込む
    ★財貨の生産が減少する

発表の要旨 (2/2) – 警告

- 銀行経済の社会福祉は、現金経済と一致。何故なら、現金に代わるものは、銀行サービス（＝要求払預金の提供）だけだから。
- 我々の単純化したモデルでは、銀行部門の存在理由を説明する新しい理論を生み出せない。
- 我々のモデルでは、財の不均質な分布は、銀行の社会福祉改善には不可欠。
- 本発表では、銀行危機を支払仲介の崩壊として説明。
- 危機から回復するために何をすべきかを解説する。

主な結果 (1/2)

銀行危機

- 基本モデル（融資強制を伴い、銀行破たんショックを伴わないモデル）
  ★銀行危機が起こらない。
- 銀行破たんショックを伴うモデル
  ★銀行危機が起こる。
- 不完全融資強制と担保制限を伴うモデル
  ★銀行破たんショックがないと仮定する。
  ★銀行危機は、預金者の自己実現型の調整の失敗の結果、起こる。

主な結果 (2/2)

- **政策的含意**
  - 財政出動 ー 政府による財貨購入

    ★政府が、購入した財貨を効果的に維持できない限り、有効ではない。
  - 金融緩和 ー 中央銀行による他銀行への融資

    ★LLR融資先が支払能力を有する銀行に限定される場合、有効ではない。
  - 銀行の支払能力を回復させる銀行改革 ー 不良債権処分と資本注入

    ★銀行預金者の自信回復と財貨の市場取引回復に有効。

    ★政策実施コストは、事前には莫大に見えても、事後には僅かであると分かる。

関連文献

- 貨幣と財貨の区別を伴う銀行モデル
  - Champ, Smith and Williamson (1996)
  - McAndrews and Roberds (1995, 1999)
  - Allen and Gale (1998)

プラン

- 基本モデル
  - 設計
  - 銀行が抱える問題
  - Night Market
  - Day Market
  - （銀行危機を伴わない）均衡
- 銀行破たんショックを伴うモデル
  - （銀行危機を伴う）均衡
- 不完全融資強制と担保制限を伴うモデル
  - 政策的含意

Basic Model – Setup (1/4)

- Closed Economy, Discrete time $t = 0, 1, \cdots, \infty$
- Two competitive markets open sequentially at each date t — Day market and Night market
- Goods:
  - Consumption goods (numeraire) — Produced in the night market
  - Intermediate goods — Produced in the day market
- Assets:
  - Machines (productive, collateralizable, last for one period)
  - Cash — Injected by Central Bank in the night market
  - Bank deposits
  - Bank loans — Not tradable

Basic Model – Setup (2/4)

- Continuum of sellers, buyers, and banks
  - Banks live for one period. Measure of banks: 1
  - Sellers live for infinite periods. Measure of sellers: n
  - Buyers live for infinite periods. Measure of buyers: $1 - n$
  - Discount factor: β (< 1) for sellers and buyers

Basic Model – Setup (3/4)

- Previous Night Market (date $t-1$): Sellers and buyers decide cash holdings, bank deposits, and bank loans that they carry over to date t.
- Day market: Anonymous market (Trade credit is not available)
  - Sellers produce and sell the intermediate goods, q, to buyers
  - Buyers have to pay cash to sellers. (Either they have cash in advance or they withdraw bank deposits)
  - After the goods trading, sellers and buyers decide cash holdings and bank deposits they carry to the night market.

Basic Model – Setup (4/4)

- Night market: Trade credit is available. Money is not needed as a medium of exchange, but is used as a store of value.
  - Buyers are endowed with machines, k. Buyers repay bank loans.
  - Sellers, buyers, (and banks) trade the intermediate goods q and machines, k
  - Buyers produce the consumption goods y from q and k by $y = Ak^{1-\theta}q^{\theta}$. Consumption takes place.
  - Bank deposits are paid out, and banks are liquidated.
  - New banks are born. Cash is injected. Cash holdings, bank deposits, and bank loans carried over to date $t+1$ are decided.

Bank's Problem (1/4)

- Banks have record keeping technology for financial transactions of sellers and buyers.
- Banks can enforce loan repayment on the borrowers.
- Date-($t-1$) night market
  - Banks make loans, L_t, hold cash reserves, C_t, and accept deposits, D_t.
- Date-t day market
  - Deposits become $(1+i_d)D_t$. Banks promise to exchange deposits to cash at anytime during the day market.
- Date-t night market
  - Banks collect loans, $(1+i)L_t$, pay out deposits, $(1+i_n)(1+i_d)D_t$, and are liquidated.

Bank's Problem (2/4)

- Banks' problem is

$$\max_{L_t, C_t, D_t} [(1+i)L_t + C_t - (1+i_n)(1+i_d)D_t]_+$$

subject to

$$L_t + C_t \leq D_t, \tag{1}$$

$$(1+i_d)D_t \leq \frac{1}{\rho} C_t. \tag{2}$$

Bank's Problem (3/4) – Cash Reserve

- The day market is divided into J submarkets. $\rho = 1/J$.
- Cash circulates J times.
  - Bank $\Rightarrow$ Buyers $j \Rightarrow$ Sellers $j \Rightarrow$ Bank $\Rightarrow$ Buyers $j+1 \Rightarrow \cdots$ (J times)
- A buyer in Buyers j withdraw all deposit: $(1+i_d)d$.
- Number of Buyers j is $(1-n)/J$.
- Total withdrawal of Buyers j: $(1+i_d)D/J$.
- Total withdrawal must equal bank's cash reserve: C
- The reserve requirement:

$$(1+i_d)D_t \leq \frac{1}{\rho} C_t.$$

Bank's Problem (4/4)

- Both (1) and (2) bind in equilibrium. The reduced form of bank's problem is

$$\max_{C_t} \left[(1+i)\left\{ \frac{1}{(1+i_d)\rho} - 1 \right\} + 1 - \frac{1+i_n}{\rho} \right]_+ C_t. \qquad (3)$$

- Since C_t cannot be infinite in equilibrium, it must be the case that

$$(1+i_d)(1+i_n) = 1 + \{1 - (1+i_d)\rho\} i, \qquad (4)$$

and the profit for the banks is zero.

Sequence of Decisions

- Date-($t-1$) night market
  - Agent chooses m^d (cash), d^d (deposit), l (loan) to carry over to the date-t day market.
- Date-t day market
  - Deposit becomes $(1+i_d)d^d$.
  - Seller produces q^s (intermediate goods) with utility cost of $c(q^s)$.
  - Buyer buys q^b units and pays pq^b.
  - Agent chooses m^n (cash), d^n (deposit) to carry over to the date-t night market. (Loan, l, does not change.)
- Date-t night market
  - Deposit becomes $(1+i_n)d^n$. Loan becomes $(1+i)l$.
  - Production, trade, and consumption of the consumption goods take place.
  - Agent chooses m^d_{+1}, d^d_{+1}, and l_{+1} to carry over to the date-($t+1$) day market.

Night Market – Seller's Problem (1/2)

- Bellman equation is

$$W^s(m^n, d^n, l) = \max_{x,h,m_{+1},d_{+1},l_{+1}} [U(x) - h + \beta V^s_{+1}(m^d_{+1}, d^d_{+1}, l_{+1})] \quad (5)$$

subject to

$$x + \phi(m^d_{+1} + d^d_{+1} - l_{+1}) = h + \phi\{m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\}, \quad (6)$$

where ϕ is the real value of cash. This program can be rewritten as

$$\begin{aligned} W^s(m^n, d^n, l) =& \phi\{m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\} \\ &+ \max_{x,m_{+1},d_{+1},l_{+1}} [U(x) - x - \phi(m^d_{+1} + d^d_{+1} - l_{+1}) + \beta V^s_{+1}(m^d_{+1}, d^d_{+1}, l_{+1})] \end{aligned}$$

Night Market – Seller's Problem (2/2)

- The first-order conditions (FOCs) are $U'(x) = 1$ and

$$\phi \geq \beta V_m^s(+1), \quad \text{where if } >, \text{ then } m_{+1}^d = 0; \text{ if } =, \text{ then } m_{+1}^d \geq 0; \tag{7}$$

$$\phi \geq \beta V_d^s(+1), \quad \text{where if } >, \text{ then } d_{+1}^d = 0; \text{ if } =, \text{ then } d_{+1}^d \geq 0; \tag{8}$$

$$\phi \leq -\beta V_l^s(+1), \quad \text{where if } <, \text{ then } l_{+1} = 0; \text{ if } =, \text{ then } l_{+1} \geq 0, \tag{9}$$

where $V_x^s(+1) \equiv \frac{\partial}{\partial x} V^s(m_{+1}^d, d_{+1}^d, l_{+1})$ for $x = m_{+1}^d, d_{+1}^d, l_{+1}$.

- The envelope conditions imply that W^s can be written as

$$W^s(m^n, d^n, l) = \phi\{m^n + (1 + i_n)d^n - (1 + i)l\} + \overline{W}_t^s, \tag{10}$$

where $\overline{W}_t^s$ is independent from the state variables.

Night Market – Buyer's Problem (1/3)

- Bellman equation is

$$W^b(q, m^n, d^n, l) = \max_{x,h,m_{+1},d_{+1},l_{+1}} [U(x) - h + \beta V^b_{+1}(m^d_{+1}, d^d_{+1}, l_{+1})] \tag{11}$$

subject to

$$x + \phi(m^d_{+1} + d^d_{+1} - l_{+1}) = h + \phi\{ak + wq + m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\}, \tag{12}$$

where k is the number of the machines, q is the quantity of the intermediate goods, and a and w are the market prices. This program can be rewritten as

$$\begin{aligned} W^s(m^n, d^n, l) =& \phi\{ak + wq + m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\} \\ &+ \max_{x,m_{+1},d_{+1},l_{+1}} [U(x) - x - \phi(m^d_{+1} + d^d_{+1} - l_{+1}) + \beta V^b_{+1}(m^d_{+1}, d^d_{+1}, l_{+1})] \end{aligned}$$

Night Market – Buyer's Problem (2/3)

- The FOCs are $U'(x) = 1$ and

$$\phi \geq \beta V_m^b(+1), \qquad \text{where if } >, \text{ then } m_{+1}^d = 0; \text{ if } =, \text{ then } m_{+1}^d \geq 0; \tag{13}$$

$$\phi \geq \beta V_d^b(+1), \qquad \text{where if } >, \text{ then } d_{+1}^d = 0; \text{ if } =, \text{ then } d_{+1}^d \geq 0; \tag{14}$$

$$\phi \leq -\beta V_l^b(+1), \qquad \text{where if } <, \text{ then } l_{+1} = 0; \text{ if } =, \text{ then } l_{+1} \geq 0. \tag{15}$$

The envelope conditions imply that W^b can be written as

$$W^b(q^b, m^n, d^n, l) = \phi\{ak + wq + m^n + (1 + i_n)d^n - (1 + i)l\} + \overline{W}_t^b, \tag{16}$$

where $\overline{W}_t^b$ is independent from the state variables.

Night Market – Buyer's Problem (3/3)

- In the night market, the buyers produce the consumption goods with the Cobb-Douglas technology, $y = Ak^{1-\theta}q^{\theta}$.
- Since k and q are competitively traded, the prices are determined by

$$\phi a = (1-\theta)A(q^b)^{\theta}, \tag{17}$$

$$\phi w = \theta A(q^b)^{\theta-1}, \tag{18}$$

since $k = 1$ and $q = q^b$ per buyer.

Day Market – Seller's Problem (1/3)

- Bellman equation is

$$V^s(m^d, d^d, l) = \max_{q, m^n, d^n} -c(q) + W^s(m^n, d^n, l) \tag{19}$$

subject to

$$m^n + d^n = pq + m^d + (1 + i_d)d^d, \tag{20}$$

$$m^n \geq 0, \text{ and } d^n \geq 0. \tag{21}$$

- This program can be rewritten as

$$V^s(m^d, d^d, l) = \max_{q, d^n} \phi pq - c(q) + \phi\{m^d + (1 + i_d)d^d + i_n d^n - (1 + i)l\} + \overline{W}_t^s$$

subject to $d^n \leq pq + m^d + (1 + i_d)d^d$.

Day Market – Seller's Problem (2/3)

- Given $i_n > 0$, the FOCs imply

$$\phi p = \frac{c'(q^s)}{1 + i_n}, \tag{22}$$

$$d^n = pq + m^d + (1 + i_d)d^d, \tag{23}$$

$$m^{ns} = 0. \tag{24}$$

- Sellers deposit all cash into their banks immediately.

Day Market – Seller's Problem (3/3)

- The envelope conditions: $V_m^s = \phi(1 + i_n)$, $V_d^s = \phi(1 + i_d)(1 + i_n)$, and $V_l^s = -\phi(1 + i)$. These conditions and the FOCs for the night market imply that

$$\phi \geq \beta\phi_{+1}(1 + i_{n,+1}), \quad \text{where if } >, \text{ then } m_{+1}^d = 0; \text{ if } =, \text{ then } m_{+1}^d \geq 0; \tag{25}$$

$$\phi \geq \beta\phi_{+1}(1 + i_{d,+1})(1 + i_{n,+1}), \quad \text{where if } >, \text{ then } d_{+1}^d = 0; \text{ if } =, \text{ then } d_{+1}^d \geq 0; \tag{26}$$

$$\phi \leq \beta\phi_{+1}(1 + i_{+1}), \quad \text{where if } <, \text{ then } l_{+1} = 0; \text{ if } =, \text{ then } l_{+1} \geq 0. \tag{27}$$

Day Market – Buyer's Problem (1/3)

- Bellman equation is

$$V^b(m^d, d^d, l) = \max_{q, m^n, d^n} \phi\{ak_t + wq + m^n + (1+i_n)d^n - (1+i)l\} + \overline{W}_t^b,$$

subject to $m^n + d^n + pq = m^d + (1+i_d)d^d$.

- In the case when $i_n > 0$,

$$d^n = m^d + (1+i_d)d^d - pq, \tag{28}$$

$$m^{nb} = 0. \tag{29}$$

- Buyers deposit all remaining money into the banks, and hold no cash.

Day Market – Buyer's Problem (2/3)

- The reduced form of the buyer's program:

$$V^b(m^d, d^d, l) = \max_q \phi\{ak + wq - (1+i_n)pq + (1+i_n)m^d + (1+i_d)(1+i_n)d^d - (1+i)l\} + \overline{W}_t^b, \tag{30}$$

subject to

$$pq \leq m^d + (1+i_d)d^d. \tag{31}$$

- The FOC is

$$(1+i_n+\lambda)p \geq w, \qquad \text{where if } >, \text{ then } q^b = 0; \text{ if } =, \text{ then } q^b \geq 0, \tag{32}$$

- The envelope conditions are $V_m^b = \phi(1+i_n+\lambda)$, $V_d^b = \phi(1+i_d)(1+i_n+\lambda)$, $V_l^b = -\phi(1+i)$, where λ is the Lagrange multiplier for (31).

Day Market – Buyer's Problem (3/3)

- The envelope conditions and the FOCs for the night market imply

$$\phi \geq \beta\phi_{+1}(1 + i_{n,+1} + \lambda_{+1}), \quad \text{where if } >, \text{ then } m^d_{+1} = 0; \text{ if } =, \text{ then } m^d_{+1} \geq 0; \quad (33)$$

$$\phi \geq \beta\phi_{+1}(1 + i_{d,+1})(1 + i_{n,+1} + \lambda_{+1}), \quad \text{where if } >, \text{ then } d^d_{+1} = 0; \text{ if } =, \text{ then } d^d_{+1} \geq 0; \quad (34)$$

$$\phi \leq \beta\phi_{+1}(1 + i_{+1}), \quad \text{where if } <, \text{ then } l_{+1} = 0; \text{ if } =, \text{ then } l_{+1} \geq 0. \quad (35)$$

Equilibrium (1/2)

- The inflation rate $\gamma = \phi/\phi_{+1}$ is determined by

$$\frac{\gamma_{+1}}{\beta} = 1 + i_{+1}. \quad (36)$$

- Since $0 < i_d < i$ and $0 < i_n < i$, sellers carry no cash nor deposit into the day market:

$$m^d = 0, \text{ and } d^d = 0. \quad (37)$$

- Liquidity constraint, (31), binds:

$$\lambda_{+1} = \rho i_{+1} > 0, \quad (38)$$

- Buyer carries no cash in the day market: $m^d = 0$. Buyer's deposit is

$$(1 + i_d)d^d = pq^b. \quad (39)$$

- $m^{nb} = d^{nb} = 0$, $d^{ns} = pq^s$, and $m^{ns} = 0$, where $q^s = (1 - n)q^b/n$.

Equilibrium (2/2)

- Buyer's purchase q^b:

$$\frac{\theta A(q^b)^{\theta-1}}{1+i} = \frac{c'(q^s)}{(1+i_d)(1+i_n)}. \tag{40}$$

- ϕd^{db} is determined by

$$\phi d^{db} = \frac{\theta A(q^b)^{\theta}}{1+i}. \tag{41}$$

- The variables for banks are determined by

$$\begin{aligned}
\phi D_t &= (1-n)\phi d^{db}, \\
C_t &= M_t. \\
\phi L_t &= \phi n l^s + \phi(1-n) l^b = (1-n)(1-\rho)\phi d^{db},
\end{aligned}$$

- $z \equiv \phi M_t = \phi C_t$ is determined by $z = (1+i_d)(1-n)\rho\phi d^{db}$.

Basic Model — No Bank Runs

- If $i_n > 0$, there are no bank runs
  - Bank insolvency never occurs. (Loan enforcement)
  - No default on bank deposits
  - Since $i_n > 0$, agents are strictly better-off by depositing their income into the banks rather than holding their income in the form of cash.
- If $i_n = 0$, bank runs may occur as a herd behavior
  - Agents are indifferent between bank deposit and cash. (The same returns)
  - Herd behavior may induce bank runs.
  - The Friedman rule is the first best. (But not so in the following model with banking crisis.)

Bank Insolvency Shock Model – Setup (1/2)

- Macroeconomic shock, $\tilde{\omega}$, hits the day market.
- Banks have complete loan enforcement technology.
- After loan repayments are made, $1 - \tilde{\omega}$ of bank assets are destroyed:

$$\tilde{\omega} = \begin{cases} 1 & \text{with probability } 1 - \delta, \\ \omega \ (< 1) & \text{with probability } \delta. \end{cases} \tag{42}$$

- If $\tilde{\omega} = \omega$, agents expect bank insolvency.
- Sellers decide to hold cash rather than deposits.
- Circulation of cash stops in the first round. (Bank runs)
  - Bank $\Rightarrow$ Buyers 1 $\Rightarrow$ Sellers 1 $\Rightarrow$ X

Bank Insolvency Shock Model – Setup (2/2)

- Each seller and buyer faces stochastic environment: $\tilde{\Gamma}$ and $\tilde{\Lambda}$
- Probability that a depositor can successfully withdraw the full amount of deposit in the day market, $\tilde{\Gamma}$.

$$\tilde{\Gamma} = \begin{cases} 1 & \text{if } \tilde{\omega} = 1, \\ \Gamma\ (< 1) & \text{if } \tilde{\omega} = \omega, \end{cases}$$

- (In Crisis, Γ is the Prob. for a buyer to be luckily in Buyers 1.)
- A depositor who holds d_t units of deposits in the night market is ultimately paid $\tilde{\Lambda} d_t$ units of cash:

$$\tilde{\Lambda} = \begin{cases} 1 & \text{if } \tilde{\omega} = 1, \\ \Lambda\ (< 1) & \text{if } \tilde{\omega} = \omega. \end{cases}$$

Bank Insolvency Shock Model

- Night Market: Optimization problems are the same as Basic Model
- Day Market: Seller's Problem
  - Bank runs do not affect (seriously) the Seller's Problem.
  - Sellers produce and sell q.
  - If $\tilde{\omega} = \omega$, sellers hold cash and do not deposit their income in the banks, anticipating a lower return on deposits, i.e., $\tilde{\Lambda} = \Lambda$ (< 1).

Day Market – Buyer's Problem

- State of a buyer: $i = n,\ s,\ f$, which occurs with probability δ_i.
  - State n: no bank run; $\delta_n = 1 - \delta$; $\tilde{\omega} = 1$
  - State s: Successful withdrawal during a bank run; $\delta_s = \delta\Gamma$; $\tilde{\omega} = \omega$.
  - State f: Failure to withdraw during a bank run; $\delta_f = \delta(1 - \Gamma)$; $\tilde{\omega} = \omega$.
- Buyer's problem is

$$V^b(m^d, d^d, l) = \sum_{i=n,s,f} \max_{q_i, m_i^n, d_i^n} \delta_i W^b(q_i, m_i^n, d_i^n, l; \Lambda_i), \qquad (43)$$

subject to budget and liquidity constraints for the respective states.

Equilibrium (1/2)

- Assume δ, probability of bank insolvency, is sufficiently small.
- It is shown that $m^{db} = 0$ (Buyers do not carry cash in the day market.)
- Buyer's problem becomes

$$\begin{aligned} V^b(m^{db}, d^{db}, l) &= \max_{q_n, q_s, q_f} E[\phi\{w_i q_i - (1+\tilde{i}_n) p_i q_i\}] + \cdots \\ &= \max_{q_n, q_s, q_f} (1-\delta)\phi\{w_n q_n - (1+i_n) p_n q_n\} + \delta\Gamma\phi\{w_\omega q_s - p_\omega q_s\} \\ &\quad + \delta(1-\Gamma)\phi\{w_\omega q_f - p_\omega q_f\} + \cdots , \end{aligned}$$

subject to

$$\begin{aligned} p_n q_n^b &\le (1+i_d) d^{db}, \\ p_\omega q_s^b &\le (1+i_d) d^{db}, \\ p_\omega q_f^b &\le 0. \end{aligned}$$

Equilibrium (2/2)

- Variables, q_n, q_s, and ϕd^d are determined by

$$c'\left(\frac{1-n}{n}q_n^b\right)q_n^b = (1+i_d)\phi d^{db},$$

$$c'\left(\frac{1-n}{n}\Gamma q_s^b\right)q_s^b = (1+i_d)\phi d^{db},$$

$$\phi d^d\left(1-\delta(1-\Gamma)\Lambda\frac{(1+i_n)(1+i_d)}{1+i}\right) = \frac{(1-\delta)\theta A(q_{n,+1}^b)^\theta + \delta\theta A(\Gamma q_{s,+1}^b)^\theta}{1+i}.$$

  The third eq. corresponds to $\phi d^{db} = \frac{\theta A(q^b)^\theta}{1+i}$ in the Basic Model.

- $\Gamma = C/\{(1+i_d)D\} = \rho$, if no cash injection.
- $\Lambda = (1+i)\omega L/(1+i_n)\{(1+i_d)D - C\} = (1-\rho+i_n)\omega/\{(1-\rho)(1+i_n)\}$, if no government guarantee.

Real Damage due to Banking Crisis

- In a banking crisis, Buyers 1 can withdraw deposits, while the other buyers (Buyers j for $j = 2, 3, \cdots, J$) cannot.
- Only Buyers 1 can purchase the intermediate goods.
- Production of the intermediate goods:
  - $(1-n)q_n^b$ in normal times
  - $(1-n)\rho q_s^b$ in the banking crisis
  - It is shown that $(1-n)\rho q_s^b < (1-n)q_n^b$.
- Production of the consumption goods:
  - $Y_n = (1-n)A(q_n^b)^\theta$ in normal times.
  - $Y_\omega = (1-n)A(\rho q_s^b)^\theta$ in the banking crisis.
  - $(Y_n - Y_\omega)/Y_n = .42$, if $\theta = 1/2$, $\rho = 1/9$, and $c(q) = q^2$.

Deflation

- Price in normal times: $\phi p_n = c' \left(\frac{1-n}{n} q_n^b \right)$
- Price in the banking crisis: $\phi p_\omega = c' \left(\frac{1-n}{n} \rho q_s^b \right)$
- Since $\rho q_s^b < q_n^b$, it is shown that $p_\omega < p_n$
- Price of the intermediate goods declines in the banking crisis.
- Lower price does not increase the demand. (Cash is necessary to buy the goods and only Buyers 1 have cash.)

Incomplete Loan Enforcement Model – Setup

- Banks cannot enforce loan repayment on the borrowers.
- Banks need to secure loans by collateral, k.
- Only buyers are endowed with k.
- Collateral constraint is

$$
\begin{aligned}
(1+i)l_t^s &= 0, && \text{for sellers} \\
(1+i)l_t^b &\leq E_{t-1}[a_t k_t], && \text{for buyers}
\end{aligned}
$$

- Macroeconomic sunspot shock, $\tilde{\omega}$, changes the depositors' expectations on the other depositors' withdrawal decision:

$$
\tilde{\omega} = \begin{cases} 1 & \text{with probability } 1-\delta, \\ \omega\ (<1) & \text{with probability } \delta. \end{cases}
$$

- If $\tilde{\omega} = \omega$, all agents believe that no sellers deposit their income in the banks.

Bank insolvency due to bank runs (1/2)

- Suppose that all agents have the expectations that all sellers never deposit their income in the banks, but hold it in the form of cash. (Bank runs)
- Agents expect that Buyers 1 can withdraw deposits and the other buyers (Buyers 2, $\cdots$, Buyers J) cannot.
- Agents expect that only Buyers 1 can buy the intermediate goods.
- Agents expect that the production of the intermediate goods decreases.

Bank insolvency due to bank runs (2/2)

- Agents expect that since the intermediate goods decrease, the marginal product of capital will decrease. ($Y = Ak^{1-\theta}q^{\theta}$.)
- Agents expect that the asset price (= MPK) will be low: $a_{\omega} = (1-\theta)A(\rho q_s^b)^{\theta}$.
  - A bank cannot enforce loan repayment.
  - When a borrower repudiates loan repayment, the bank can only seize the collateral, $k\ (=1)$, and sell it at the price of a.
  - If $(1+i)l^b > a_{\omega}$, the banks cannot collect the full amount of bank loans.
  - Bank assets in the night market become $(1-n)a_{\omega} < (1+i)L$.
- Agents expect that the banks become insolvent once bank runs occur.

Coordination Failure

- If $\tilde{\omega} = 1$, agents expect the other agents deposit their income immediately in the banks (No bank runs)
  - Prodction and trading in the day market are normally done.
  - Asset price will be a_n ($> (1+i)l^b$).
  - Banks will be solvent.
  - Optimal decision for sellers and buyers is to hold bank deposits. (No-bank-run expectation is justified.)
- If $\tilde{\omega} = \omega$, agents expect the other agents to never deposit their income in the banks (Bank runs)
  - Production and trading in the day market are disrupted.
  - Asset price will be a_ω ($< (1+i)l^b$).
  - Banks will be insolvent.
  - Optimal decision is to hold cash (Bank-run expectation is justified)

Equilibrium

- Equilibrium is calculated just like that of the Bank Insolvency Shock Model.
- Only difference is the endogeneity of Λ:
  - Bank asset in the night market becomes $(1-n)a_\omega k$.
  - Bank liability becomes $(1+i_n)\{(1+i_d)D - C\} = (1+i_n)(1+i_d)(1-\rho)(1-n)d^{db}$.
  - The value of Λ is determined by

$$\Lambda = \frac{(1-n)a_\omega k}{(1+i_n)(1+i_d)(1-\rho)(1-n)d^{db}} = \frac{(1-\theta)A\rho^\theta (q_s^b)^\theta}{(1+i_n)(1+i_d)(1-\rho)\phi d^{db}}.$$

政策的含意 (1/4)

- **金融政策（LLR融資）**
  - 中央銀行は、他銀行にその資産価値 $(1-n)a\omega$ まで現金を融資する。銀行は現金準備を$C+(1-n)a\omega$まで増やすことができるが、それは、預金者（＝買い手）により引き出され得るものである。
  - この政策は、Day Marketでの取引を促進する。
  - 通常の生産まで回復させるには不十分。
    - ★必要な現金: $(1+id)D-C$
    - ★LLR融資: $(1-n)a\omega$
    - ★$(1+id)D-C>(1-n)a\omega$
  - LLR融資後の資産価格aLは、$a\omega$より高い。しかし、$(1+i)L>(1-n)aL$ である。
  - 銀行は依然として破たん寸前。銀行への預金取り付けは継続。
  - 生産と中間財取引の崩壊により、依然として財の損失が生じている。

政策的含意 (2/4)

- **銀行改革は銀行の支払能力を回復する。**
  - 政府は、銀行システムの支払能力の回復を保証し補助金を出す。（例：一律保証、強制的資産評価の後に行う資本注入）
  - 銀行の支払能力が回復すると、売り手は皆、Day Marketで収益を銀行に預金する。（預金の利益率は、現金の利益率を厳密には上回る。）
  - 銀行は現金準備不足に陥らなくなる。
  - 銀行への預金取り付けは終息し、通常の中間財生産が回復する。
  - 政策実施コストはゼロ。

    ★資産価格がanまで上昇し、$(1-n)an > (1+i)L$を満たす。

    ★銀行は、貸付金の完全回収能力を回復する。銀行資産は$(1+i)L + C$となる。

    ★銀行は公的資金注入なしで支払能力を回復する。

政策的含意 (3/4)

- **財政政策**
- **ケース1：政府が中間財を適切に維持できる。**
  - 政府は、銀行危機の間に中間財を購入し、Night Marketで売却する。
  - 中間財と消費財の生産が回復する。
  - 資産価格は*an*まで上昇する。銀行の支払能力が回復する。
  - 銀行への預金取り付けは終息する。
  - 社会厚生は改善し、政策実施コストはゼロである。
  - これは最適政策である。

政策的含意 (4/4)

- **財政政策**
- **ケース2：政府は中間財を適切に維持できない。**
  - 政府は銀行危機の間に中間財を購入するが、購入した財はDay Marketの間に消滅する。
  - 中間財の生産は回復するが、消費財の生産は回復しない。
  - 資産価格は$a\omega$にとどまる。銀行は依然として破たん寸前である。
  - 銀行への預金取り付けは終息しない。
  - 政策実施コストは莫大で、税が投入されなければならない。
  - 社会厚生は改善されない。厚生は買い手から売り手に再分配されない。

現在の金融危機への含意

- 財政出動　ー　政府の財貨購入
  - 政府が、購入した財貨を効果的に維持・活用できない限り、有効ではない。
  - 恐らく、危機の更なる悪化を食い止められない。
- 金融緩和　ー　中央銀行による他銀行への融資
  - LLR融資先が支払能力を有する銀行に限定される場合、有効ではない。
  - 恐らく、銀行への預金取り付け（もしくは優良品への資本逃避）をとめられない。
- 銀行の支払能力を回復させる銀行改革　ー　不良債権処分と資本注入
  - 銀行負債における信用回復と財貨の市場取引回復に有効。
  - 資産価格は、政策の意図するように動く。銀行は公的資金注入なしで支払能力を回復する。
  - 政策コストは、事前には莫大に見えても、事後、最終的には僅かであると分かる。

雑 記

- モデルの発展的拡大： 生産性ショックと景気循環の組み込み
- 固有ショックと個々の銀行預金取り付け
- 個々の銀行預金取り付けの悪影響